Corrections to "Energy-Efficient Controller Design for a Redundantly-Actuated Hybrid Feed Drive With Application to Machining"

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THIS note corrects an error in our paper [1], which presented energy-efficient feed-forward and feedback controller designs for a redundantly actuated hybrid feed drive using an optimal control input ratio. It shows that the optimal control input ratio derived in [1] is missing an adjoint operator. Furthermore, it shows that the correct optimal ratio (i.e., the ratio with the adjoint operator) is noncausal, hence difficult to implement in practice. Finally, it demonstrates that the ratio derived in [1] is a causal approximation of, and an appropriate substitute for, the correct ratio in the cases studied in [1].

Maintaining the same notations as in [1], the condition for energy optimality derived in [1, eq. (7)] is repeated as

$$\int \left(\frac{2u_{ff1}}{{K_{m1}}^2} \delta u_{ff1} + \frac{2u_{ff2}}{{K_{m2}}^2} \delta u_{ff2} \right) dt = 0. \tag{1}$$

This functional, combined with [1, eq. (5)], leads to

$$\begin{aligned} &\int \left(\tfrac{2u_{ff1}}{{K_{m1}}^2} \left(-{G_{21}}^{-1} G_{22} \delta u_{ff2} \right) + \tfrac{2u_{ff2}}{{K_{m2}}^2} \delta u_{ff2} \right) dt = 0 \\ &\Rightarrow \left({G_{21}}^{-1} G_{22} \right)^* u_{ff1} = \kappa^2 u_{ff2} \end{aligned} \tag{2}$$

where $\kappa = K_{m1}/K_{m2}$ and the superscript $*$ indicates the adjoint operator, which is missing in [1, eq. (8)]. The absence of the adjoint operator in [1, eq. (8)] implies that the system dynamics is self-adjoint, which is not generally true for linear systems.

Based on the corrected expression in (2), the correct optimal control input ratio for the feed-forward control is given by

$$u_{ff1} = \beta^* u_{ff2} \tag{3}$$

where $\beta = \kappa^2 G_{22}^{-1} G_{21}$ is the optimal control ratio defined in [1, eq. (25)], which is missing the adjoint operator ($*$). Note that, even though β^* guarantes optimality, it is noncausal [2], and thus requires solving a boundary value problem (offline) to be useful for the controller design approaches described in [1]. On the other hand, β is causal, and thus is convenient for the real time implementation, as demonstrated in [1]. In the rest of this note, we show that, for the hybrid feed drive studied in [1], β is an appropriate substitute for β^*, in terms of energy optimality; hence it is suitable for the energy efficient controller design presented in [1].

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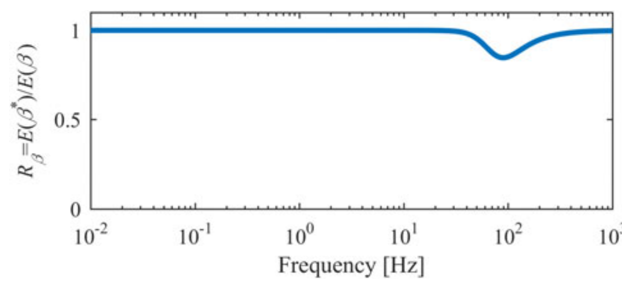

Fig. 1. Energy efficiency ratio R_β as a function of frequency.

Let us use η to describe an arbitrary ratio between u_{ff1} and u_{ff2}. In accordance with [1, eq. (5)], the two feed-forward control signals are uniquely calculated as

$$\begin{cases} u_{ff1} = \eta u_{ff2} \\ x_d = G_{21} u_{ff1} + G_{22} u_{ff2} \end{cases} \Rightarrow \begin{cases} u_{ff1} = \eta (G_{21}\eta + G_{22})^{-1} x_d \\ u_{ff2} = (G_{21}\eta + G_{22})^{-1} x_d \end{cases}. \tag{4}$$

Due to Parseval's theorem, the integral of the time domain energy cost functional defined in [1, eq. (6)] can be expressed in the frequency domain. Accordingly, the integrand at each frequency is given by

$$\begin{aligned} E(\eta, \omega) &\triangleq \frac{u_{ff1}(j\omega)\, {u_{ff1}}^*(j\omega)}{{K_{m1}}^2} + \frac{u_{ff2}(j\omega)\, {u_{ff2}}^*(j\omega)}{{K_{m2}}^2} \\ &= \left. \frac{x_d {x_d}^* \left(\eta\eta^* + \kappa^2 \right)}{{K_{m1}}^2 (G_{21}\eta + G_{22})(G_{21}\eta + G_{22})^*} \right|_{s=j\omega}. \end{aligned} \tag{5}$$

Therefore, the frequency-dependent energy efficiency ratio R_β, between the correct ratio (i.e., $\eta = \beta^*$) and its causal approximation (i.e., $\eta = \beta$), is given by

$$\begin{aligned} R_\beta(\omega) &= \frac{E(\beta^*, \omega)}{E(\beta, \omega)} \\ &= \left. \frac{G_{21}{G_{21}}^*\beta\beta^* + G_{21}{G_{22}}^*\beta + {G_{21}}^*G_{22}\beta^* + G_{22}{G_{22}}^*}{G_{21}{G_{21}}^*\beta\beta^* + G_{21}{G_{22}}^*\beta^* + {G_{21}}^*G_{22}\beta + G_{22}{G_{22}}^*} \right|_{s=j\omega}. \end{aligned} \tag{6}$$

Note that $R_\beta \approx 1$, when $|\beta(j\omega)| \gg 1$, $|\beta(j\omega)| \ll 1$, or $\angle\beta(j\omega) \approx n\pi\ (n \in \mathbb{Z})$. Therefore, the deviation of R_β from unity is limited to certain frequency ranges based on these conditions.

The impact of using β in [1] instead of β^* is investigated using R_β, calculated with the hybrid feed drive parameters provided in [1, Table I]; the result is shown in Fig. 1. At most frequencies $R_\beta \approx 1$, the only exception is the region around 100 Hz, where R_β deviates significantly from unity (because $\beta(j\omega)$ violates the magnitude and phase relationships required to keep $R_\beta \approx 1$). Nonetheless, in the worst case, R_β only drops to 85% at 89 Hz.

Therefore, the overall efficiency cost of using the approximate ratio β instead of the correct ratio β^* is small.

Considering the practical benefits gained from its causality combined with its near-optimality, we conclude that the approximate control ratio derived in [1] is very suitable for the energy-efficient controller design, as presented in [1]. Note that the same correction, analysis, and conclusion apply to β derived for the feedback design in [1, eqs. (24) and (25)].

References

[1] M. Duan and C. Okwudire, “Energy-efficient controller design for a redundantly-actuated hybrid feed drive with application to machining,” *IEEE/ASME Trans. Mechatronics*, vol. 21, no. 4, pp. 1822–1834, Aug. 2016.

[2] R. W. Beard, “Linear operator equations with applications in control and signal processing,” *IEEE Control Syst. Mag.*, vol. 22, no. 2, pp. 69–79, Apr. 2002.