

Evolving stress-guided continuous fiber path optimization for multi-axis additive manufacturing of composite shells

Yuexin Yang^a, Ziyi Xu^a , Molong Duan^{a,b,*}

^a Department of Mechanical and Aerospace Engineering, The Hong Kong University of Science and Technology, Hong Kong Special Administrative Region of China

^b HKUST Shenzhen-Hong Kong Collaborative Innovation Research Institute, Futian, Shenzhen, China

ARTICLE INFO

Keywords:

Continuous fiber path optimization
Toolpath planning
Multi-axis additive manufacturing
Shell structures

ABSTRACT

Continuous fiber-reinforced polymer composites exhibit superior mechanical properties critical for engineering applications demanding high strength-to-weight ratios. Additive manufacturing enables customized fiber placement and fabrication of complex geometries, particularly for shell structures. Existing research mostly exploits the stress fields in homogeneous components and optimizes fiber layouts by aligning the fibers with the principal stress fields. However, fiber incorporation substantially alters stress distribution, potentially resulting in suboptimal fiber path design without considering the evolution of stress fields across successive layers. Additionally, existing methodologies typically formulate the fiber optimization as a direct 3D problem, thereby encountering significant challenges when simultaneously addressing spatial fiber layouts, stress field considerations, fiber reinforcement effects, and manufacturing constraints. This work presents a novel, efficient framework for the design and manufacturing of mechanically optimized multi-layer composite shells through fiber path optimization strategies. The 3D shell surface is initially mapped to a 2D domain using modified quasi-conformal mapping, with a derived semi-analytical relationship characterizing distortion and expansion. Within the 2D domain, the component's stress field is computed iteratively, and the evolving stress distributions guide a multi-objective optimization incorporating density-controllable path extraction algorithms to generate continuous fiber paths. Furthermore, a constrained one-stroke toolpath generation algorithm is formulated to address manufacturing limitations while ensuring fiber continuity. Apart from the effectiveness of the proposed method shown by simulation results, experimental validation through multi-axis additive manufacturing confirms that optimized structures achieve enhanced structural stiffness and increased peak load capacity compared to conventional constant spacing and orientation fiber patterns, as well as existing stress-guided fiber optimization approaches.

1. Introduction

Continuous-fiber-reinforced polymers (CFRP) are widely used in the aerospace, automotive, and other industries due to their lightweight nature, high stiffness, and strength [1]. Conventional manufacturing processes—including pultrusion, vacuum forming, resin transfer molding, and vacuum bagging—require tooling and specialized equipment that limit geometric complexity and design flexibility [2]. These processes typically yield CFRP components with uniform fiber orientations (e.g., 0°, ±45°, 90°) and consistent fiber volume fractions [3]. In contrast, additive manufacturing (AM) enables the fabrication of geometrically intricate structures without molds or extensive tooling,

while simultaneously supporting automation [4]. In particular, extrusion-based CFRP-AM allows the deposition of continuous, steered fibers along freeform trajectories on customizable build platforms [5]. This capability facilitates the development of tailored designs with spatially varying and directionally optimized mechanical properties that can be precisely controlled at the local level [6–8].

Extrusion-based CFRP-AM still encounters various printing defects such as fiber misalignments, abrasion, detachment, among others [9]. Process optimization represents a fundamental approach for enhancing CFRP performance through systematic improvement of manufacturing quality [5]. Process monitoring and control have demonstrated effectiveness in enhancing mechanical properties [10]. Lu et al. [11] utilized

* Corresponding author at: Department of Mechanical and Aerospace Engineering, The Hong Kong University of Science and Technology, Hong Kong Special Administrative Region of China.

E-mail address: duan@ust.hk (M. Duan).

<https://doi.org/10.1016/j.addma.2025.105012>

Received 20 August 2025; Received in revised form 13 October 2025; Accepted 31 October 2025

Available online 6 November 2025

2214-8604/© 2025 Published by Elsevier B.V.

vision-based in-situ monitoring to detect defects during AM processes and adjusted layer height to improve fiber deposition accuracy. Yang et al. [12] exploited contact force feedback for closed-loop control by dynamically adjusting printing parameters (e.g., fiber feed rate, printing velocity) to enhance printing quality, particularly in large curvature regions. Additionally, researchers have employed supplementary mechatronic hardware and energy sources, including laser treatment [13] and an additional pressure roller [14] to address printing quality challenges. Nonetheless, the improvement in mechanical performance through process optimization alone is limited, prompting a shift toward structural design optimization to fully exploit the potential of CFRPs.

Structural optimization seeks to improve mechanical performance while minimizing weight, cost, and material usage through the adjustment of shape, size, and material layout under prescribed loading and boundary conditions. Finite element analysis (FEA) is central to this process, providing stress distributions needed for informed design decisions. Due to the anisotropic and heterogeneous nature of CFRPs, several modeling strategies have been developed, including the orthotropic model [15,16], representative volume element (RVE) model [17,18], and membrane-embedded model [19,20]. Among these, the orthotropic model offers a favorable trade-off between accuracy and computational efficiency by homogenizing fiber-matrix behavior into orthotropic finite elements [16,21,22], making it particularly suitable for thin-walled shell structures.

For fixed-geometry components, infill path optimization has emerged as a powerful technique to exploit CFRP anisotropy. When continuous fibers align with principal stress trajectories, mechanical performance can be significantly enhanced [23], especially when fiber density is positively correlated with stress magnitude [24]. Stress-guided (SG) fiber path generation methods are typically categorized into explicit and implicit approaches. Explicit methods determine fiber paths through direct control of ordered point sequences. Wang et al. [25] determined fiber paths via stress vector tracing, enabling paths to follow load transmission routes. He et al. [22] utilized B-splines to define fiber paths and optimize orientation through gradient-based methods to achieve superior strength-to-weight ratios. While explicit methods provide superior path control accuracy, they require substantial computational resources, particularly for high infill densities involving numerous fiber paths. On the other hand, implicit methods utilize a scalar field to represent the fiber paths, which is constructed by discrete control points in the design domain. By optimizing the scalar field, the fiber paths are extracted through isolines, and the fiber infill density is efficiently controlled by extracting heights. The extracted isolines undergo path planning to generate continuous fiber toolpaths with minimized trajectory length and cutting points, facilitating manufacturing processes [26]. Li et al. [7] applied mapping-based methods for infill density optimization. Liu et al. [27] proposed wave projection functions to shape stress-aligned infill morphologies. These SG optimization methods demonstrated superior mechanical properties compared to conventional uniform filling strategies with constant spacing and orientation (CSO) paths. However, traditional layer-by-layer AM restricts fiber deposition to planar directions, constraining performance optimization in complex 3D geometries.

Multi-axis AM using robotic arms introduces additional degrees of freedom, enabling freeform fiber paths across non-planar surfaces. Focusing on developable surfaces, Zhang et al. [28] generated conformal fiber toolpaths and successfully printed fiber ribs crossing shell surfaces using robot-assisted AM. For solid models, Fang et al. [6] proposed non-planar slicing strategies optimizing 3D stress-based scalar fields and extracting curved layers for printable spatial infill fiber path generation. For shell structures, Liu et al. [8] employed geodesic distance to precisely represent distances between adjacent infill toolpaths, enabling 3D stress-guided path generation with infill density control on shell surfaces. These approaches embed fibers in 3D space, significantly enhancing mechanical properties compared to planar counterparts.

Despite these advances, most SG methods optimize fiber paths based

on the stress field of homogeneous parts without considering the fiber reinforcement influence. Since fiber deposition alters the structure's stiffness and stress distribution, neglecting this feedback can lead to suboptimal designs. Hou et al. [29] adopted a discretization-based method to assign fiber orientations and distributions within each element based on the nearest fiber track point along the extracted path, which improves accuracy at high resolution but incurs considerable computational cost. To address this, He et al. [22] introduced a semi-analytical fiber-in-element probability (AFEP) model for efficient approximation of fiber-structure interaction. Incorporating this feedback has been shown to improve performance, though prior work is largely limited to 2D domains with sparse fiber layouts. Recent approaches have begun integrating SG with topology optimization (TO), enabling concurrent optimization of macro-scale topology and micro-scale fiber trajectories. Ren et al. [21] proposed a unified formulation in which elemental pseudo-densities (for topology) and scalar field gradients (for toolpaths) are co-optimized. To ensure manufacturability, Fernandez et al. [30] incorporated material anisotropy and process constraints directly into the optimization, while Luo et al. [31] regulated the curl and divergence of fiber orientation fields using an Augmented Lagrangian formulation to enforce bend radius and overlap limits. However, optimization within a single layer can not always achieve the optimal steered fiber paths design, due to the manufacturing constraints such as minimum path spacing and allowable curvature. Several studies have designed a thickness-variable CFRP shell with curvilinear fiber tapes to mitigate the high stress region by locally expanding the fiber volume along the thickness direction [32,33].

In multi-layer laminated composite shells, fiber infill paths often vary across different layers. Conventional CSO designs employ cross-layered patterns (e.g., $\pm 45^\circ$) [3], whereas recent SG methods use principal stress directions to define layer-specific paths, naturally producing orthogonal reinforcement [8,26]. To further enhance performance, more diverse, freeform fiber paths across layers have been explored. However, a significant gap remains in efficiently designing multi-layer composite shells that simultaneously consider: (1) stress evolution due to fiber reinforcement, (2) geometric and manufacturing constraints inherent to extrusion-based AM, and (3) computational feasibility for spatially complex 3D surfaces. To address these challenges, this work proposes a computationally efficient, sequential layer-by-layer evolving-stress-guided optimization framework for multi-layer CFRP shell design. Unlike iterative global optimization approaches that aim for convergence across the entire structure, the proposed method sequentially optimizes each layer based on the updated stress field reflecting the influence of previously deposited fibers. This progressively coupled strategy accommodates practical manufacturing constraints (e.g., minimum curvature, limited fiber volume per layer), enabling variable-stiffness and variable-stiffness design. The main contributions of the paper are:

- A bijective, quasi-conformal mapping between a 3D shell surface and a 2D parametric domain is established, enabling computationally efficient fiber path design by leveraging optimization within the simplified 2D space.
- Guided by an evolving stress field, an iterative, scalar-field-based multi-objective optimization framework is developed, complemented by a density-controllable method for extracting fiber paths from the optimized fields.
- A three-dimensional semi-analytical fiber-in-element probability function is introduced to capture the reinforcement effects of extracted fiber paths within each shell layer, thereby generating an evolving stress field.
- A one-stroke fiber deposition toolpath generation method is proposed, considering multiple manufacturing constraints, which facilitates experimental validation through multi-axis CFRP-AM case studies.

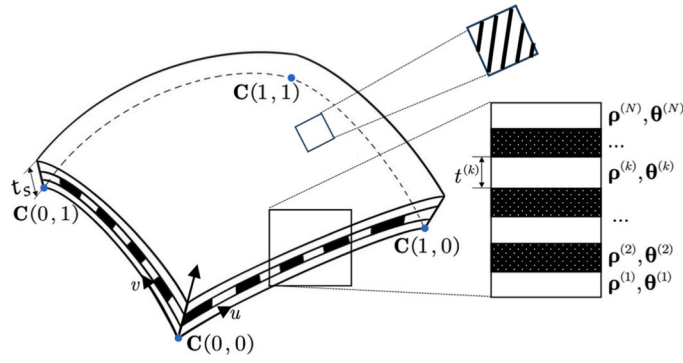


Fig. 1. Multilayer composite shell structure with its parameterization.

The detailed paper is structured as follows: The evolving stress-guided (ESG) fiber optimization and density-controlled fiber extraction procedure within the simplified 2D space is detailed and derived in Section 2. The one-stroke fiber deposition toolpath and multi-axis printing trajectory generation methods are detailed in Section 3. In Section 4, the proposed ESG optimization method is systematically compared with the CSO and SG methods through both FEM simulations and experimental validation using a robot-based multi-axis printing platform, followed by conclusions in Section 5.

2. Iterative fiber path optimization for composite shells

2.1. Compliance minimization problem over a shell structure

Shell structures are characterized by their thin-walled geometry derived from surfaces. For a shell structure with a thin thickness t_s , its surface geometry is quasi-equivalent for each layer. To simplify the problem, the bottom layer of the shell structure is extracted as a representative layer for parameterization, as shown in Fig. 1. Consider a curved surface $\mathbf{C}(u, v) : [0, 1] \times [0, 1] \rightarrow \mathbb{R}^3$, where u and v are normalized parameters defining the surface geometry within an orthogonal curvilinear coordinate system in the parametric space. The vertices $\mathbf{C}(0, 0)$, $\mathbf{C}(1, 0)$, $\mathbf{C}(1, 1)$ and $\mathbf{C}(0, 1)$ define the four corner points of the parametric domain. The surface topology is determined by the boundary conditions: when the edges $\mathbf{C}(0, v)$ and $\mathbf{C}(1, v)$ are not connected, the shell exhibits an open configuration. Conversely, when $\mathbf{C}(0, v) = \mathbf{C}(1, v)$, the structure forms a closed surface, with $\mathbf{C}(0, v)$ constituting the developing line. Composite laminated shells comprise a N number of layers, each layer $k \in \mathbb{Z}$ is deposited sequentially along the surface normal direction with a thickness $t^{(k)}$, contributing incrementally to the overall shell thickness t_s .

To achieve optimal structural performance in shell topology design with enhanced stiffness, the minimization of structural compliance under a prescribed material volume constraint serves as a fundamental formulation in topology optimization [34–36]. In the context of multilayered shell structures, the design space is enriched with additional degrees of freedom, enabling the variation of fiber paths and material distribution across individual layers. Finite element modeling (FEM) is employed by discretizing the continuous shell surface $\mathbf{C}(u, v)$ into triangulations along u and v directions, forming a set $\mathcal{S}$ consisting of N_e elements and N_n nodes. An orthotropic material model is adopted to represent the composite behavior of each element, wherein the combined effect of fiber and matrix is homogenized based on the local fiber volume fraction and orientation. For the CFRP material fabricated via CFRP-AM, the fiber volume fraction within each element is approximated as proportional to the local material density, while the fiber orientation coincides with the deposition orientation of the CFRP.

Consequently, the objective of this study is to determine the optimal distribution of CFRP material density $\rho \in \mathbb{R}^{N_e} \times \mathbb{R}^N$ and orientation $\theta \in \mathbb{R}^{N_e} \times \mathbb{R}^N$ at the element level across all layers, such that the resulting

structure exhibits maximum stiffness (minimum compliance) while satisfying a global constraint on total volume of CFRP materials. The corresponding optimization problem is thus mathematically formulated as follows:

$$\begin{aligned} \min_{\rho} c &= \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^E \mathbf{u}_e^T \mathbf{K}_e(\rho_e, \theta_e) \mathbf{u}_e, \\ \text{Find : } \rho &= \{\rho^1, \rho^2, \dots, \rho^N\}, \theta = \{\theta^1, \theta^2, \dots, \theta^N\}, \\ \text{s.t. } &\begin{cases} \mathbf{K} \mathbf{U} = \mathbf{F} \\ \mathbf{K}_e = A_e t^{(k)} \mathbf{B}_e^T \mathbf{D}_e(\rho_e, \theta_e) \mathbf{B}_e \\ V_f = \sum_{k=1}^N \sum_{e=1}^E A_e \rho_e^{(k)} t^{(k)} \leq V_{\max}, \\ 0 \leq \rho_e^{(k)} \leq 1 \\ 0 \leq \theta_e^{(k)} \leq \pi \end{cases} \end{aligned} \quad (1)$$

where c is the compliance of the whole structure, which is obtained from the global displacement matrix $\mathbf{U}$, stiffness matrix $\mathbf{K}$, and force vector matrix $\mathbf{F}$; $\mathbf{u}_e$ is the element displacement matrix and $\mathbf{K}_e$ is the element stiffness matrix. For an element e with area A_e and thickness $t^{(k)}$, its material density ρ_e and orientation θ_e determine the constitutive matrix $\mathbf{D}_e$ of CFRP material. $\mathbf{B}_e$ is element shape function matrix. The total material volume V_f is subjected to the constraint $V_{\max}$. The ρ and θ across total N layers are treated as design variables, governed by the geometry and density of the underlying fiber paths. Consequently, the design of a multilayer composite shell with minimized compliance is equivalent to the problem of optimizing the spatial distribution of fiber paths across multiple layers. Accordingly, the Eq. (1) defines the global optimization objective of this study. To achieve this objective, a comprehensive computational framework is developed, which includes bijective surface mapping, evolving stress field generation, scalar field optimization, and fiber reinforcement estimation, as detailed in the subsequent sections.

2.2. Bijective mapping of shell surfaces with local direction and length transformation relationships

The fiber orientation within each layer significantly influences the mechanical performance of the composite structure. Direct generation of 3D fiber paths for each layer presents substantial computational challenges, particularly for complex geometries. This is due to the high dimensionality of design variables when points are defined in 3D Cartesian space. Moreover, incorporating manufacturing constraints—such as fiber spacing, curvature limits, and turning angle thresholds—directly in 3D space is relatively abstract and computationally intensive. In computational geometry, a surface is conceptualized as a two-dimensional manifold with intrinsic non-zero curvature embedded in three-dimensional Euclidean space. According to the Riemann mapping theorem, most simply connected domains can be mapped conformally in a bijective manner onto the open unit disk in the complex plane [37]. Conformal mappings preserve local angular relationships, thereby enabling the design of fiber orientations within a 2D parametric domain while ensuring geometric consistency when mapped back to the original 3D surface. Moreover, the computational cost associated with performing conformal mapping is negligible. This dimension reduction significantly enhances the efficiency of spatial fiber path optimization by reducing computational complexity, a strategy that has proven effective in structural optimization, such as graded infill design [38] and rib-reinforced thin-walled structures [39] on 3D surfaces. Accordingly, this study establishes a bijective conformal mapping framework that accounts for both local directional fidelity and length distortions. The 3D shell surface is first parameterized onto a 2D computational domain, where multi-objective optimization of fiber paths is performed. The

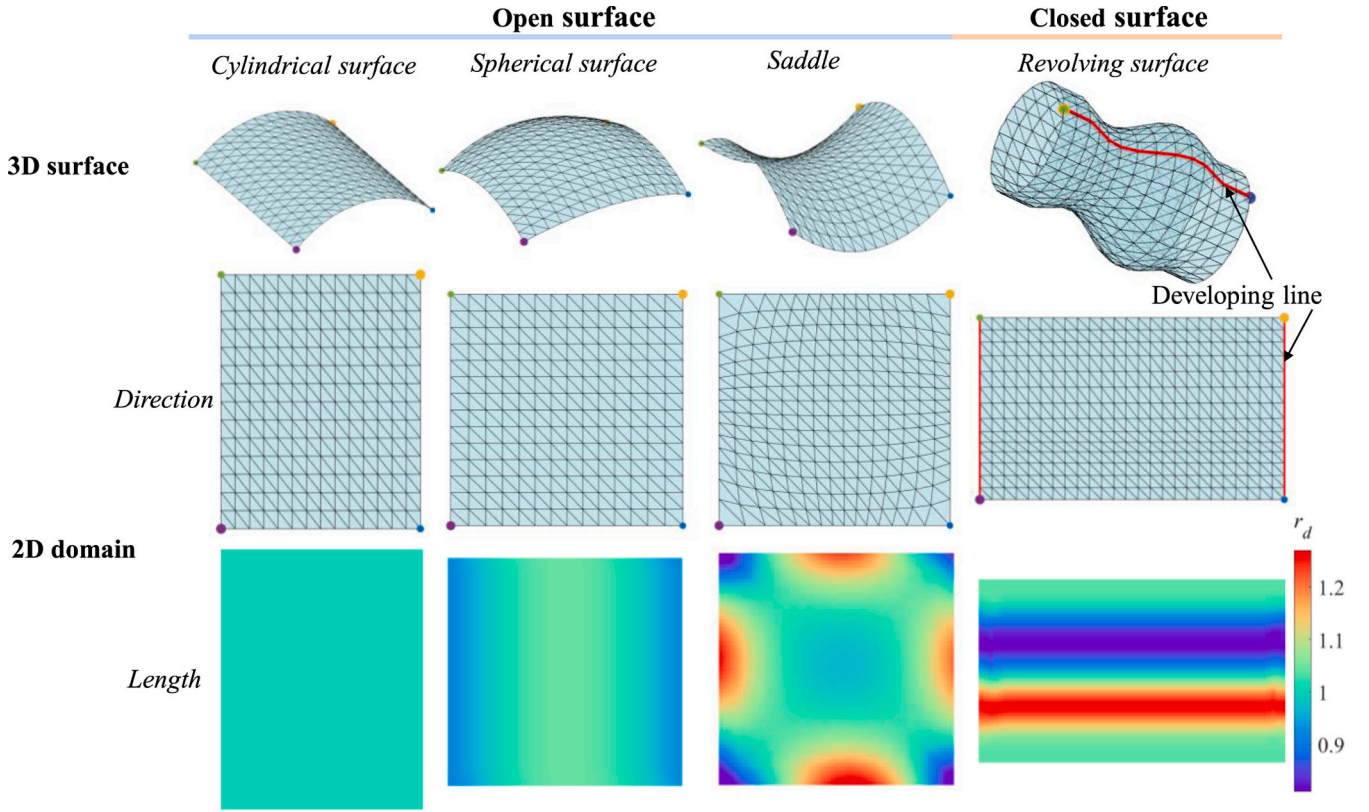


Fig. 2. Bijective mapping of different-shaped shell surfaces from 3D to 2D and corresponding local direction and length transformation maps.

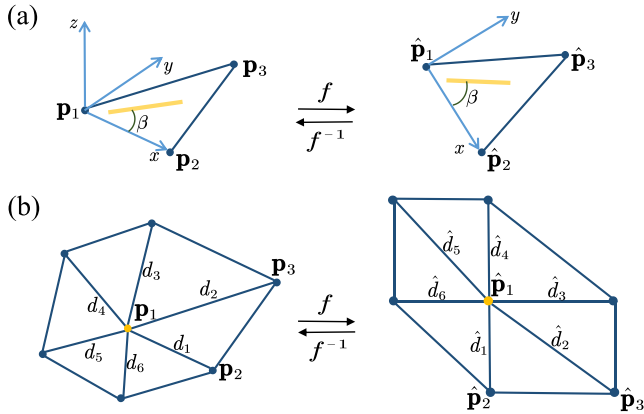


Fig. 3. Transformation relationship of (a) a vector in an element, and (b) local length distortion for a node during bijective mapping.

optimized fiber trajectories are then transformed back to the 3D geometry via inverse mapping, enabling accurate and efficient realization of fiber-reinforced designs on complex shell structures through optimization conducted in a simplified 2D domain.

Rectangular domains offer superior formulation properties and enhanced convenience for path generation algorithms compared to alternative geometries such as unit disks. The landmark-constrained Teichmüller mapping algorithm [40] provides an effective approach for mapping 3D surfaces onto rectangular planes. The landmarks constraints refer to the specific feature points (e.g., corners, feature markers, or user-defined correspondences) on the original surfaces that are mapped directly to their prescribed target landmark points in the 2D domain, ensuring the important geometric features, like boundary corners, align correctly between two shapes during mapping. This

quasi-conformal mapping is incorporated with landmark constraints with minimized angular distortion, thereby preserving fiber reinforcement characteristics along prescribed directions while maintaining corresponding geometric features. To establish the quasi-conformal mapping parameterization between 3D shells and 2D planes, the shell is triangulated into domain $\mathcal{S}$. The triangular elements comprise discrete nodal points represented by $\mathbf{S} = (x, y, z)$. The landmark-constrained mapping relationship $f: \mathcal{S} \rightarrow \mathcal{P}$ is expressed as

$$\begin{aligned} (u, v) &= f(x, y, z), \\ \text{s.t. } f(\mathbf{p}_{s,i}) &= \mathbf{p}_{t,i}, i = 1, 2, \dots, \end{aligned} \quad (2)$$

where $\hat{\mathbf{S}} = (u, v)$ is transformed points on the 2D domain $\mathcal{P}$ and f^{-1} represents its inverse mapping. The source landmark on 3D shell is $\mathbf{p}_{s,i}$ and its target landmark is $\mathbf{p}_{t,i}$. The quasi-conformal mapping for different surface types from a 3D surface $\mathcal{S}$ to a 2D domain $\mathcal{P}$ is shown in Fig. 2. During the mapping procedure, the four vertices of open shells are set as landmarks and are mapped to corresponding vertices in the parametric domain, marked in colored points in Fig. 2. For closed shells, the lateral surface is treated as an open surface connected by a developing line, with the two endpoints of the developing line serving as landmark constraints mapped to the four corners of the two-dimensional parametric rectangular plane.

Quasi-conformal mappings preserve angles between distinct intersecting curves at each point of intersection, while typically inducing distortion in their lengths. To achieve a convincing 3D fiber path design in a simplified 2D domain, variations of direction and length during mapping need to be evaluated quantitatively. As illustrated in Fig. 3(a), for a triangular element e on the 3D surface, its three nodes $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ are connected in the counterclockwise direction. The normal vector of the surface at the triangle e is calculated by

$$\mathbf{N}(e) = \frac{(\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)}{\|(\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)\|}. \quad (3)$$

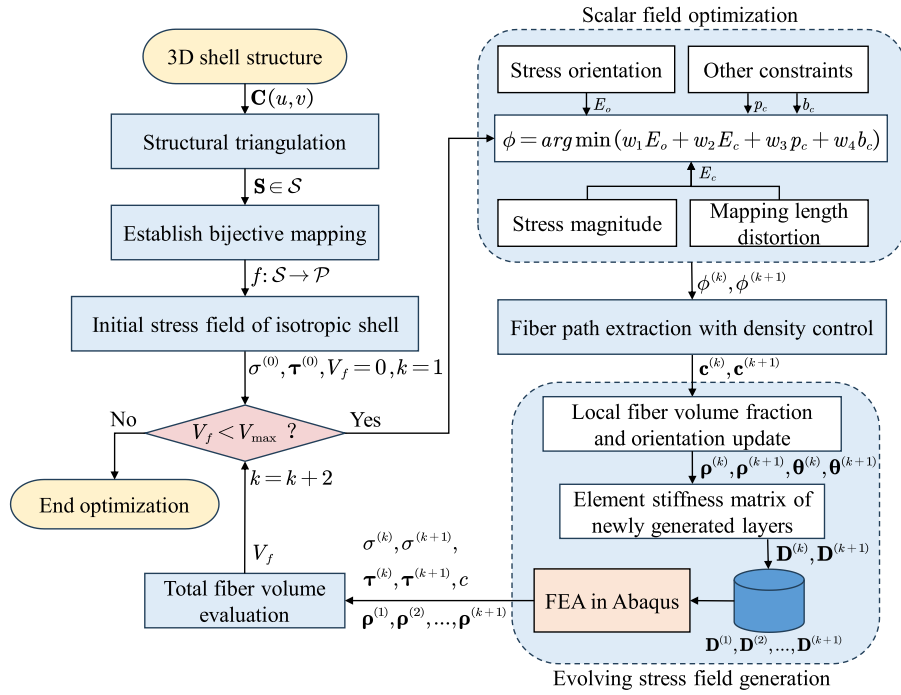


Fig. 4. Evolving stress field guided iterative fiber path optimization procedure.

Define the vector $\mathbf{v}_1(e) = (\mathbf{p}_2 - \mathbf{p}_1) / \|\mathbf{p}_2 - \mathbf{p}_1\|$ as x axis in its local coordinate system, the normal vector $\mathbf{N}(e)$ defined in Eq. (3) is z axis, the vector $\mathbf{v}_2(e)$ forms y axis, and is obtained by

$$\mathbf{v}_2(e) = \mathbf{v}_1(e) \times \mathbf{N}(e). \quad (4)$$

Then, the local coordinate system for the 3D element e is formulated and given by

$$\mathbf{M}_3 = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{N}]. \quad (5)$$

Similarly, the triangular mesh mapped on 2D domain has three nodes $\hat{\mathbf{p}}_1$, $\hat{\mathbf{p}}_2$, and $\hat{\mathbf{p}}_3$. Define a normalized vector $\hat{\mathbf{p}}_{12} = (\hat{\mathbf{p}}_2 - \hat{\mathbf{p}}_1) / \|\hat{\mathbf{p}}_2 - \hat{\mathbf{p}}_1\|$, the x axis in this elements is $\hat{\mathbf{v}}_1(e) = [\hat{\mathbf{p}}_{12} \ 0]^\top$. The normal vector $[0, 0, 1]^\top$ which points out of the element plane donates z axis, and the vector $\hat{\mathbf{v}}_2(e)$ forms y axis, given by

$$\hat{\mathbf{v}}_2 = \hat{\mathbf{v}}_1 \times [0, 0, 1]^\top. \quad (6)$$

The local coordinate system for a 2D element e is thus established by

$$\mathbf{M}_2 = \begin{bmatrix} \hat{\mathbf{v}}_1 & \hat{\mathbf{v}}_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (7)$$

Then the transformation matrix is formulated by

$$\mathbf{M} = \mathbf{M}_3^{-1} \mathbf{M}_2, \quad (8)$$

and a vector $\mathbf{v}_3$ in the 3D surface is transformed to a vector $\mathbf{v}_2$ by the following equation:

$$\mathbf{v}_2 = \mathbf{M} \mathbf{v}_3, \quad (9)$$

which maintains the angle β between the element edge and the vector consistently during mapping, as shown in Fig. 3(a).

Most 3D surfaces are non-developable, necessitating stretching or compression during planar mapping. Although quasi-conformal mappings preserve angular relationships, they introduce metric distortion, resulting in local length distortion factors $r_d(u, v)$. When a 2D domain $\mathcal{P}$

is obtained through conformal mapping f from $\mathcal{S}$, discretized into triangular elements with nodal connectivity, its distortion ratio r_d for each vertex node n is approximated by the following equation [41]:

$$r_d(n) = \frac{\sum_{i=1}^3 d_i}{\sum_{i=1}^3 \hat{d}_i}, \quad (10)$$

where d_i and $\hat{d}_i$ are the length of adjacent edges from the original triangulation on the 3D surface and the distorted triangulation on the 2D plane, respectively (Fig. 3(b)). The r_d is easily converted to a continuous function $r_d(u, v)$ via interpolation and $1/r_d$ represents the inverse mapping relationship. Fig. 2 illustrates the 2D length distortion map for different-shaped surfaces. The r_d value for a developable cylindrical surface is a constant value while other non-developable surfaces, such as spherical surfaces and saddles, have varying r_d at different locations. This local length distortion factor is incorporated into the scalar field optimization to compensate for 2D-3D mapping distortions, thereby preserving both fiber orientation and density on the 3D surface.

2.3. Evolving stress-guided (ESG) iterative fiber path optimization

To design a composite shell comprising multiple layers, FEA is conducted on the 3D shell under specified loading conditions to obtain the stress distribution and guide fiber path design. To optimize the reinforcement of anisotropic fibers in a composite shell, the fiber path direction θ in each layer is designed to align with the principal stress directions, while the fiber volume fraction ρ is distributed in proportion to the corresponding principal stress magnitudes. Once a layer of fibers is introduced based on an initial stress field, the material stiffness matrix $\mathbf{D}$ changes, thereby changing stress distribution under the same loading conditions. To account for this evolving stress field with the addition of optimized layers, an iterative fiber path optimization methodology is employed for each successive layer, so that each newly placed layer achieves optimal fiber alignment to best accommodate the current stress state. The whole procedure of this optimization is illustrated in Fig. 4.

To start with, a homogeneous shell with isotropic material is defined to obtain the initial stress field $\sigma^{(0)}$ and $\tau^{(0)}$. The optimization is con-

ducted in a simplified 2D domain via the bijective mapping established in Section 2.2; thereby, the consideration of length distortion during mapping and other manufacturing constraints is considered in optimization. In each iteration k , an FEA is conducted on the shell structure with the predefined loading condition to obtain the stress fields. The $\alpha^{(k)}$ and $\tau^{(k)}$ represent the value and orientation of the maximum principal stress, respectively; $\alpha^{(k+1)}$ and $\tau^{(k+1)}$ represent the value and orientation of the minimum principal stress, respectively. The maximum and minimum principal stress fields are exploited to guide fiber path optimization with scalar field $\phi^{(k)}$ and $\phi^{(k+1)}$, respectively, resulting in the addition of two quasi-orthogonal layers. Then, the fiber paths are extracted from the optimized scalar field, and their fiber volume fraction $\rho^{(k)}$, $\rho^{(k+1)}$ and orientation $\theta^{(k)}$, $\theta^{(k+1)}$ are estimated to influence the stiffness matrix $\mathbf{D}^{(k)}$, $\mathbf{D}^{(k+1)}$. Subsequently, an FEA is performed using an updated anisotropic stiffness matrix that reflects the newly introduced fiber layers. The fiber volume V_f of the resulting structure with $k+1$ layers is compared against a predefined constraint $V_{\max}$. If the fiber volume remains below the threshold, the optimization proceeds by introducing additional fiber layers guided by the resulting evolving stress field $\sigma^{(k)}$, $\sigma^{(k+1)}$, $\tau^{(k)}$, $\tau^{(k+1)}$. This iterative process continues until the total fiber volume exceeds the constraints with minimized compliance.

2.3.1. Evolving stress field

In the iterative optimization process, as the number of composite layers increases, the resulting stress field changes, called the evolving stress field. For the multi-layer 3D composite shell comprising k layers, its evolving stress tensors $\mathbf{T}_s^{(k)}(e)$ for each element e is obtained from FEA results. The principal stresses $[\sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)}]$ and their corresponding directions $[\tau_1^{(k)}, \tau_2^{(k)}, \tau_3^{(k)}]$ are calculated through eigenvalue decomposition of $\mathbf{T}_s^{(k)}(e)$. The maximum principal stress $\sigma_{\max}^{(k)} = \sigma_1^{(k)}$, minimum principal stress, and their corresponding principal stress directions $\tau_{\max}^{(k)}$ and $\tau_{\min}^{(k)}$ are obtained by sorting the absolute values of stresses in descending order. The $\tau_{\max}^{(k)}$ and $\tau_{\min}^{(k)}$ are perpendicular vectors defined in the local coordinate system in each element. Knowing the local coordinate system of each element $\mathbf{M}_0$, the maximum and minimum principal stress directions in the global coordinate system are calculated by

$$\begin{aligned} \tau_{\max 0}^{(k)} &= \mathbf{M}_0 \tau_{\max}^{(k)}, \\ \tau_{\min 0}^{(k)} &= \mathbf{M}_0 \tau_{\min}^{(k)}. \end{aligned} \quad (11)$$

With Eq. (9), the stress direction in the 2D plane becomes

$$\begin{aligned} \hat{\tau}_{\max}^{(k)} &= \mathbf{M} \tau_{\max 0}^{(k)}, \\ \hat{\tau}_{\min}^{(k)} &= \mathbf{M} \tau_{\min 0}^{(k)}. \end{aligned} \quad (12)$$

The stress value and direction are discrete values and are approximated to be located at the center position of each element e . Since the physical orientation of stress is inherently directionless (i.e., indistinguishable under 0° and 180° rotations), the vector directionality must be removed. The resulting stress orientation field and stress value are then interpolated onto a regular and continuous function, denoted $\sigma^{(k)}(u, v)$ and $\tau^{(k)}(u, v)$.

2.3.2. Scalar field optimization

To efficiently represent the fiber paths for the k -layer shell, a scalar function $\phi^{(k)}(u, v)$ defined in the 2D design domain is established. The fibers are generated by extracting contours of $\phi^{(k)}$ at different heights, and their directions are consistent with contour directions. The problem of optimizing fiber path explicitly becomes to optimize $\phi^{(k)}$ implicitly. The contours of $\phi^{(k)}$ are aligned with the target stress field $\tau^{(k)}$, meaning the gradient direction of $\phi^{(k)}$ are perpendicular to $\tau^{(k)}$, which derives the following criterion to be minimized:

$$E_o = \int_{\mathcal{D}} \|\nabla \phi^{(k)} \cdot \tau^{(k)}\|^2. \quad (13)$$

In addition to fiber orientation, the material density constitutes a critical design parameter requiring optimization. Regions experiencing stress concentrations necessitate higher fiber volume fractions to provide adequate local reinforcement and ensure structural integrity. Conversely, regions subjected to lower stress levels can accommodate reduced material density, thereby achieving material efficiency while minimizing structural mass without compromising performance requirements. The absolute value of stress $|\sigma^{(k)}|$ is converted to reference material density $\rho_r^{(k)}$ by linear mapping and given by

$$\rho_r^{(k)} = \rho_{\min} + (\rho_{\max} - \rho_{\min}) \frac{|\sigma^{(k)}| - \min(|\sigma^{(k)}|)}{\max(|\sigma^{(k)}|) - \min(|\sigma^{(k)}|)}, \quad (14)$$

where $\rho_{\max}$ and $\rho_{\min}$ are the maximum and minimum material density, respectively. The gradient value of $\phi^{(k)}(u, v)$ represents the intervals of fiber arrangement, and the gradient value of $\phi^{(k)}(u, v)$ is proportional to the reference material density $\rho_r^{(k)}$, which derives the following criterion to be minimized:

$$E_r = \int_{\mathcal{D}} (\|\nabla \phi^{(k)}\| - \rho_r^{(k)})^2. \quad (15)$$

To equivalently and accurately optimize 3D fiber paths on a 2D domain, the local length distortion of the mapped domain should be taken into consideration. The gradient magnitude of the objective function is consistent with local length distortions, and its error is defined by

$$E_d = \int_{\mathcal{D}} (\|\nabla \phi^{(k)}\| - r_d)^2. \quad (16)$$

The E_r and E_d both contribute to the fiber interval arrangement. To simplify the problem, they are combined to E_c , given by

$$E_c = \int_{\mathcal{D}} (\|\nabla \phi^{(k)}\| - r^{(k)})^2, \quad (17)$$

$r^{(k)} = \rho_r^{(k)} r_d$.

For closed-shell structures, the process of flattening a 3D surface into a 2D open plane introduces a developing line, which corresponds to a single continuous edge in 3D but becomes two separate boundary curves in the 2D domain. During fiber path optimization on this 2D plane, it is critical to ensure the fiber orientations along these two boundary curves remain as consistent as possible. This consistency ensures that, upon mapping the 2D layout back onto the original 3D geometry, the fiber paths align smoothly across the developing line. To quantitatively enforce this boundary continuity, a dedicated objective term is introduced to measure and minimize the mismatch in fiber orientation along the corresponding boundary curves.

$$b_c = \int_{\mathcal{D}} \left(\frac{\nabla \phi^{(k)}(0, v)}{\|\nabla \phi^{(k)}(0, v)\|} - \frac{\nabla \phi^{(k)}(W, v)}{\|\nabla \phi^{(k)}(W, v)\|} \right)^2, \quad (18)$$

where W is the width of the 2D domain $\mathcal{D}$. When extracting contours from an optimized scalar function $\phi^{(k)}$, the formation of closed contours should be avoided. Closed contours are challenging to integrate with other fiber paths, thereby increasing the complexity of generating a continuous, one-stroke printing path for continuous fiber additive manufacturing. The emergence of closed contours is typically associated with local convex or concave regions of the scalar field $\phi^{(k)}$. These extremal points can be readily identified, and their numbers are penalized as variable $p_c \in \mathbb{Z}$ during the optimization process. Combining the Eqs. (13), (17), and (18), the total optimization setup is formulated as

$$\phi^{(k)} = \operatorname{argmin}(w_1 E_o + w_2 E_c + w_3 p_c + w_4 b_c), \quad (19)$$

Table 1

Contour-based fiber path extraction from scalar function.

Input: scalar function $\phi^{(k)}$, fiber width f_w , density factor K_d , distortion ratio r_d , step size Δh
Output: fiber paths $\mathbf{c}^{(k)}$
Main:
// initialization
$h = \max(\phi^{(k)}) - \Delta h$
$\mathbf{c}_1^{(k)} = \text{contour}(\phi^{(k)}(u, v) = h)$
$i = 2$
$\mathbf{c}^{(k)} = \mathbf{c}_1^{(k)}$
while $h > \min(\phi^{(k)})$ do
$j = 1$
$d_{\min} = 0$
//Density control by K_d
while $d_{\min} \leq K_d f_w$ do
// update contour height
$h = h - j \Delta h$
$\mathbf{c}_i^{(k)} = \text{contour}(\phi^{(k)}(u, v) = h)$
$\mathbf{d}_0 \leftarrow$ Euclidean distance of point pair $[\mathbf{c}_i^{(k)}, \mathbf{c}^{(k)}]$
$\mathbf{d}_c = \mathbf{d}_0(r_d(\mathbf{c}_i^{(k)}) + r_d(\mathbf{c}^{(k)}))/2$
$d_{\min} = \min(\mathbf{d}_c)$
$j = j + 1$
end
$\mathbf{c}^{(k)} = \{\mathbf{c}_1^{(k)}, \mathbf{c}_2^{(k)}, \dots, \mathbf{c}_i^{(k)}\}$
$i = i + 1$
end

where w_1 , w_2 , w_3 , and w_4 are the weights for each variable.

The scalar function $\phi^{(k)}(u, v)$ defined over a rectangular domain is easily represented using a 2D non-uniform rational B-spline (NURBS) surface parameterization. The NURBS surface is constructed from a grid of control points $\mathbf{P} = [\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z]^\top$, which are uniformly distributed across the domain based on its physical dimensions, given by

$$\Phi(\xi_u, \xi_v) = \sum_{i=0}^{n_u} \sum_{j=0}^{n_v} N_{i,m_u}(\xi_u) N_{j,m_v}(\xi_v) \mathbf{P}_{ij}, \quad (20)$$

where $\xi_u, \xi_v \in [0, 1]$ are the B-spline curve parameters, n_u, n_v are the number of control points, m_u, m_v define the order of B-splines. Surface $\Phi(\xi_u, \xi_v)$ is converted to $\phi^{(k)}(u, v)$ by interpolation. The $\mathbf{p}_x$ and $\mathbf{p}_y$ are fixed and $\mathbf{p}_z$ is treated as design variables. The values of $\mathbf{p}_z$ equal zero at the beginning and are varied to govern the shape of the scalar field. Consequently, the optimization of the scalar function $\phi^{(k)}(u, v)$ is formulated as the optimization of the values $\mathbf{p}_z$ of control points, with Eq. (19) converted to:

$$\mathbf{p}_z = \text{argmin}(w_1 E_o + w_2 E_c + w_3 p_c + w_4 b_c), \quad (21)$$

where objective E_o governs the fiber orientation, objective E_c controls the material density, penalty p_c ensures manufacturability, and boundary condition b_c is applied to the closed shell to improve fiber continuity at boundaries. Multi-objectives are incorporated with different optimization weights, and weight values are selected based on the magnitude of each individual term and optimization tendency.

2.3.3. Density-controlled fiber path extraction

The varying-height contours of a scalar function $\phi^{(k)}(u, v)$ represent the distribution of fiber paths. Contour height intervals between con-

tours directly determine fiber path density, which subsequently governs the structural mechanical properties. Table 1 outlines the methodology for extracting fiber paths from a scalar function while maintaining density control. The extraction process initiates at the maximum value of $\phi^{(k)}(u, v)$ and proceeds downward in decrements defined by a specified step size Δh in each iteration. The contour extracted at the current height h constitutes the fiber path $\mathbf{c}_i^{(k)}$, which is represented by a set of discrete points sampled at small spatial intervals and ordered along the contour. The curve set $\mathbf{c}^{(k)} = \{\mathbf{c}_1^{(k)}, \mathbf{c}_2^{(k)}, \dots, \mathbf{c}_{i-1}^{(k)}\}$ denotes the collection of previously extracted fiber paths. Geometrically, for two non-intersecting curves, the spacing between them can be evaluated by computing the Euclidean distance between points sampled along each curve. Let $\mathbf{p}_a$ be a point on the current curve $\mathbf{c}_i^{(k)}$ and a point $\mathbf{p}_b$ on an existing curve $\mathbf{c}_j \in \mathbf{c}^{(k)}$; the distance between them is given by $\|\mathbf{p}_a - \mathbf{p}_b\|$. During each iteration, the pairwise Euclidean distances $\mathbf{d}_0 \in \mathbb{R}^{N_a} \times \mathbb{R}^{N_b}$ are computed between all discrete points on the current fiber path $\mathbf{c}_i^{(k)}$ and those on the aggregate set of existing curves $\mathbf{c}^{(k)}$, where N_a and N_b denote the number of discrete points sampled on $\mathbf{c}_i^{(k)}$ and $\mathbf{c}^{(k)}$, respectively.

Since the fiber path extraction algorithm operates in a 2D parametric domain, and to account for potential length distortions upon mapping to the 3D surface, a corrected distance $\mathbf{d}_c$ is introduced. This corrected distance is defined as

$$\mathbf{d}_c = \mathbf{d}_0 \frac{r_d(\mathbf{c}_i^{(k)}) + r_d(\mathbf{c}^{(k)})}{2}, \quad (22)$$

where $r_d(\cdot)$ denotes the local distortion factor. This formulation allows the 2D Euclidean distance to serve as an efficient approximation of the true 3D distance between points after surface mapping. Given that the fiber filament employed in this study has a width value $f_w < 1\text{ mm}$, when $\mathbf{d}_c$ approaches f_w , the difference between the Euclidean and geodesic

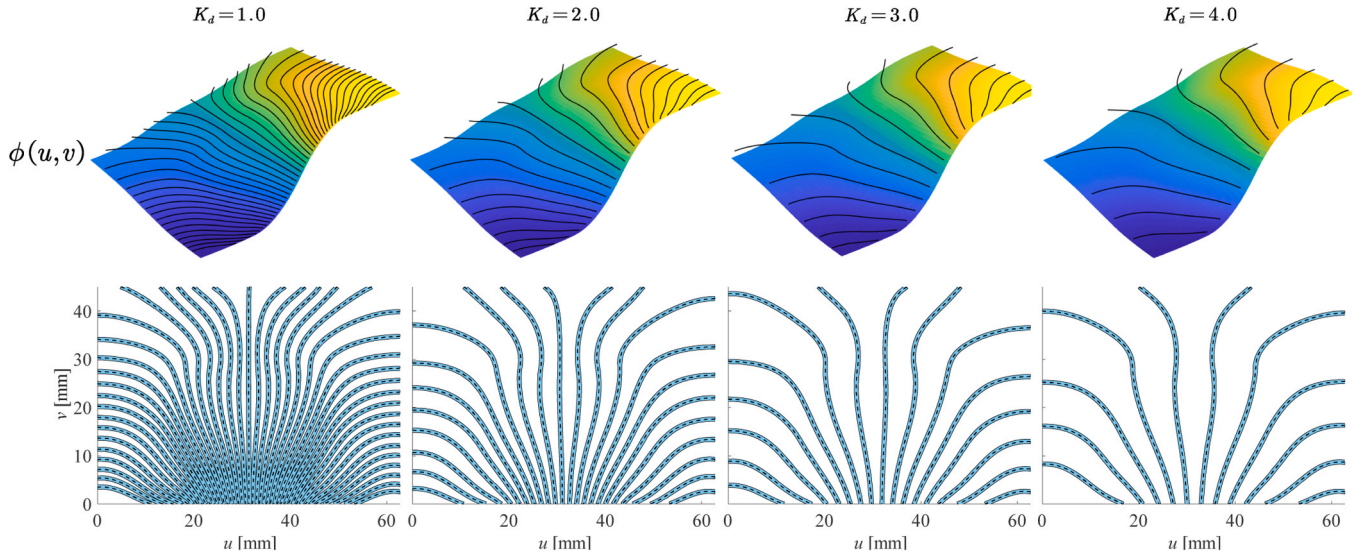


Fig. 5. Extracted fiber segments from an optimized scalar field with density factors K_d .

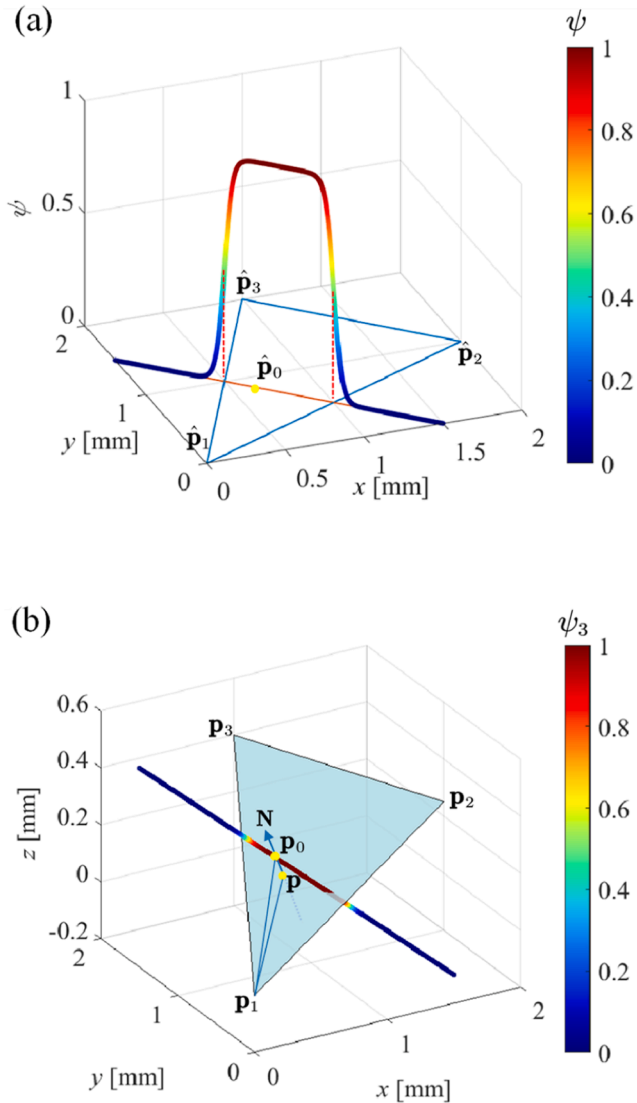


Fig. 6. Schematics of (a) 2D and (b) 3D AFEP function.

distances becomes negligible due to local surface flatness. The minimum corrected distance $d_{\min}$ is used to enforce the spacing constraint between adjacent fiber paths. Only candidate path $\mathbf{c}_i^{(k)}$ that satisfies this constraint are incorporated into the set of existing paths $\mathbf{c}^{(k)}$.

To optimize mechanical properties, achieving maximum fiber density is desirable. Manufacturing limitations necessitate maintaining a minimum allowable separation between adjacent fibers to prevent overlapping. The density factor K_d is then introduced to regulate the minimum spacing between neighboring fiber paths. When K_d equals one, the composite shell achieves maximum material density with dense fiber arrangement. As K_d increases, the fiber path spacing expands proportionally, resulting in reduced fiber density across the shell surface. Fig. 5 demonstrates fiber segments extracted from an optimized scalar function guided by maximum principal stress directions, where K_d varies from 1.0 to 4.0 from left to right, illustrating the progressive reduction in fiber density within the formed composite layer.

The fiber extraction procedure continues iteratively until the current h reaches the minimum value of $\phi^{(k)}$, signifying complete traversal with all qualifying fiber paths successfully extracted and incorporated into the final set $\mathbf{c}^{(k)}$.

2.3.4. Fiber reinforcement contribution

The generated fiber paths affect the constitutive matrix via the fiber density and orientation inside the corresponding element and further influence the mechanical properties of the whole structure. Inspired by [22], a semi-analytical fiber-in-element probability (AFEP) function defined in a 2D domain describing the intersection relationship is adopted and defined as

$$\psi(\hat{\mathbf{p}}_0, \hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2, \hat{\mathbf{p}}_3) = \prod_{j=1}^3 \sigma_{\text{sig}} \left(\frac{(\hat{\mathbf{p}}_0 - \hat{\mathbf{p}}_j) \times (\hat{\mathbf{p}}_{j+1} - \hat{\mathbf{p}}_j)}{f_w \|\hat{\mathbf{p}}_{j+1} - \hat{\mathbf{p}}_j\|} \right), \quad (23)$$

where $\psi(\hat{\mathbf{p}}_0, \hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2, \hat{\mathbf{p}}_3)$ approximates the probability of an arbitrary point $\hat{\mathbf{p}}_0$ inside the 2D triangular element with 3 nodes $\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2, \hat{\mathbf{p}}_3$. The binary relationships are replaced by the multiplication of the sigmoid function σ_{sig} to indicate probability. The sigmoid function σ_{sig} is defined as

$$\sigma_{\text{sig}}(\xi) = \frac{1}{1 + e^{-\xi}}, \quad (24)$$

which approaches one and zero as the variable tends towards positive and negative infinity, respectively. This property, combined with the

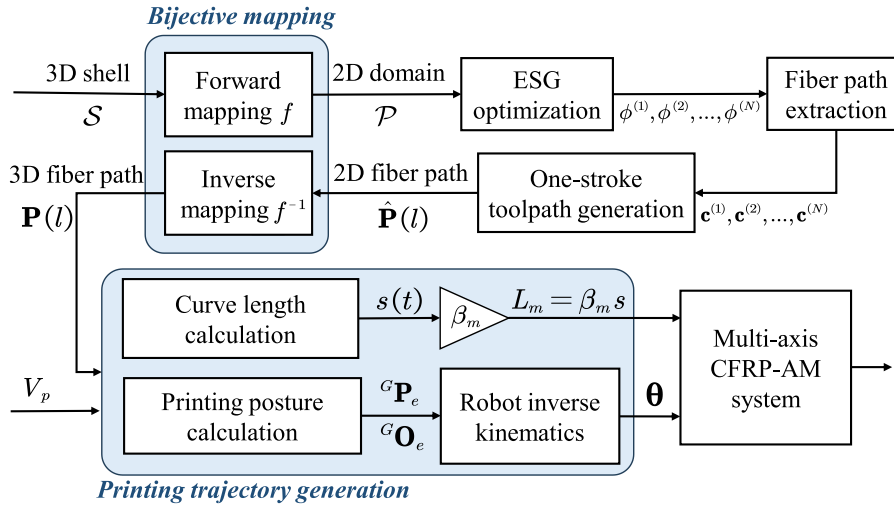


Fig. 7. One-stroke toolpath and printing trajectory generation for multi-axis printing.

normalization by the edge length $\|\hat{\mathbf{p}}_{j+1} - \hat{\mathbf{p}}_j\|$ and the fiber width f_w , ensures that $\psi = 1$ when the point $\hat{\mathbf{p}}_0$ is at least a half-width from the boundary, and $\psi = 0.5$ when the point $\hat{\mathbf{p}}_0$ sits on the boundary of the element (Fig. 6(a)). This continuous probability distribution corresponds directly to the physical intersection area between the fiber and finite element, thereby enhancing the computational smoothness of fiber reinforcement evaluation. When a point $\hat{\mathbf{p}}_0$ is mapped to a 3D surface, given as $\mathbf{p}_0$, due to the triangulation accuracy limit, the point is not always located in the mesh plane, as shown in Fig. 6(b). The point is firstly projected into the mesh plane and given by

$$\mathbf{p} = \mathbf{p}_0 - \frac{(\mathbf{p}_0 - \mathbf{p}_1) \cdot \mathbf{N}}{\mathbf{N} \cdot \mathbf{N}} \mathbf{N}, \quad (25)$$

where $\mathbf{N}$ is the element normal vector defined in Eq. (3). Then, the 3D AFEP function (Fig. 6(b)) is derived to evaluate the fiber-mesh intersection relationship more accurately and is shown as

$$\tilde{\psi}(\tilde{\mathbf{c}}^{(k)}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \begin{cases} \prod_{j=1}^3 \sigma_{\text{sig}} \left(\frac{((\tilde{\mathbf{c}}^{(k)} - \mathbf{p}_j) \times (\mathbf{p}_{j+1} - \mathbf{p}_j)) \cdot \mathbf{N}}{f_w \|\mathbf{p}_{j+1} - \mathbf{p}_j\|} \right) & \psi \neq 0 \\ 0 & \psi = 0 \end{cases}, \quad (26)$$

where $\tilde{\mathbf{c}}^{(k)}$ is set of 3D fiber paths mapped from $\mathbf{c}^{(k)}$, which is previously extracted with Table 1. The length of i th fiber inside the e th element is given by

$$L_{i,e}^{(k)} = \int \tilde{\psi}(\tilde{\mathbf{c}}_i^{(k)}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \left\| \frac{d\mathbf{c}_i^{(k)}}{dl} \right\| dl. \quad (27)$$

The CFRP volume fraction contributed by i th fiber in element e is calculated by

$$\rho_{i,e}^{(k)} = \frac{L_{i,e} f_w}{A_e}, \quad (28)$$

where A_e is the area of element e . The total fiber volume ratio for the element e is then derived by

$$\rho_e^{(k)} = \min \left(\rho_{\max}, \sum_{i=1}^{N_f} \rho_{i,e}^{(k)} \right), \quad (29)$$

where $\rho_{\max}$ is the CFRP volume fraction threshold and N_f is the number of fibers. The $\rho_{\max}$ is introduced as a practical constraint to ensure both manufacturability and physical realism. Although fiber spacing is

explicitly controlled in the fiber path extraction algorithm (as detailed in Section 2.3.3) to prevent fiber overlap and to maintain feasible deposition paths, the discretization of the domain into finite elements can lead to localized overestimation of the cumulative volume fraction. This occurs when multiple fiber paths intersect the same element, potentially resulting in artificially elevated values of $\rho_e^{(k)}$. To address this and to promote smoothness in the estimated volume fraction field $\rho_e^{(k)}$ across neighboring elements, an additional smoothing step is applied. Specifically, after enforcing the upper bound on the local CFRP volume fraction as defined in Eq. (29), the resulting field is convolved with a 2D Gaussian kernel of size $n_g \times n_g$. The resulting smoothed field is then mapped back to the original mesh elements, preserving the resulting field $\rho^{(k)}$ satisfies the required physical constraints. The i th fiber direction is obtained by

$$\mathbf{T}_{f,i}^{(k)} = \frac{\mathbf{c}_i^{(k)}}{\|\mathbf{c}_i^{(k)}\|}. \quad (30)$$

A representative fiber orientation within each finite element is determined using a weighted averaging scheme guided by the AFEP function, which accounts for the smoothness and spatial distribution of fiber trajectories, thereby enabling an efficient and accurate approximation of the local orientation, expressed as:

$$\mathbf{T}_{f0,e}^{(k)} = \sum_{i=1}^{N_f} \left(\frac{1}{L_{i,e}} \int \tilde{\psi}(\tilde{\mathbf{c}}_i^{(k)}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \mathbf{T}_{f,i}^{(k)} \left\| \frac{d\mathbf{c}_i^{(k)}}{dl} \right\| dl \right), \quad (31)$$

$$\mathbf{T}_{f,e}^{(k)} = \mathbf{T}_{f0,e}^{(k)} / \|\mathbf{T}_{f0,e}^{(k)}\|.$$

The vector $\mathbf{T}_{f,e}^{(k)} = [z_x^{(k)}, z_y^{(k)}]$ defines the average fiber direction $\theta_e^{(k)}$ inside this element e , and $\theta_e^{(k)}$ is given by

$$\theta_e^{(k)} = \arctan(z_y^{(k)} / z_x^{(k)}). \quad (32)$$

The isotropic matrix material is defined with Young's modulus E_m and Poisson ratio μ , the elastic coefficient matrix $\mathbf{D}_m$ is given by

$$\mathbf{D}_m = \frac{E_m}{1 - \mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1 - \mu}{2} \end{bmatrix}. \quad (33)$$

Similarly, for the fiber-reinforced material, the longitudinal and transverse Young's moduli are defined as E_1 and E_2 , respectively, the in-plane Poisson ratios are defined as μ_{12} and $\mu_{21} = \mu_{12} E_2 / E_1$, the shear modulus

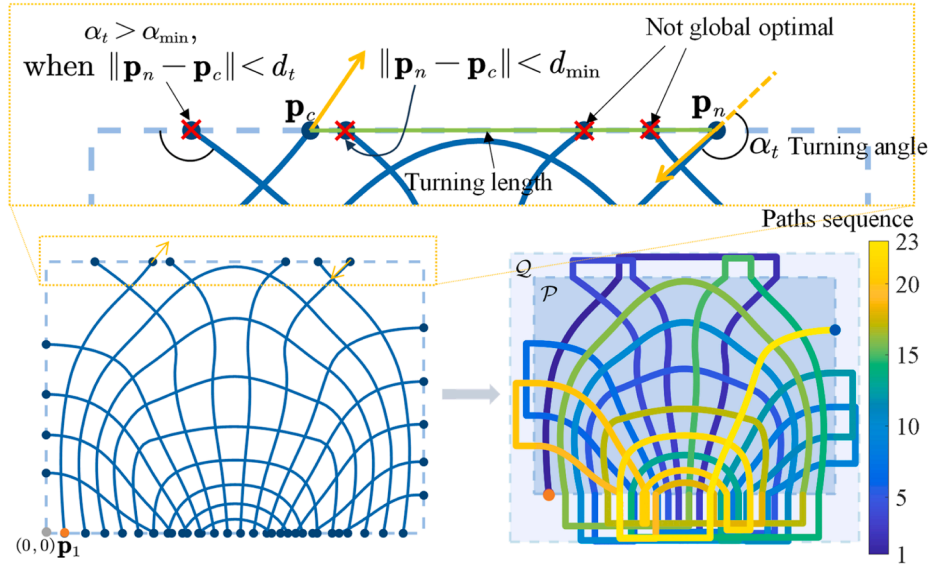


Fig. 8. Schematics of fiber segment connection with manufacturing constraints and path sequence illustration of one-stroke toolpath.

is G_{12} . The 2D anisotropic element elastic matrix $\mathbf{D}_{f0}$ is given by

$$\mathbf{D}_{f0} = \begin{bmatrix} \frac{E_1}{1 - \mu_{12}\mu_{21}} & \frac{\mu_{12}E_2}{1 - \mu_{12}\mu_{21}} & 0 \\ \frac{\mu_{12}E_2}{1 - \mu_{12}\mu_{21}} & \frac{E_2}{1 - \mu_{12}\mu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}. \quad (34)$$

For a composite element with CFRP material density and orientation $\theta_f^{(k)}$, the constitutive matrix $\mathbf{D}_e^{(k)}$ is estimated as

$$\mathbf{D}_e^{(k)} = \mathbf{T}(\theta_e^{(k)})^\top (\rho_e^{(k)} \mathbf{D}_{f0}) \mathbf{T}(\theta_e^{(k)}). \quad (35)$$

where $\mathbf{T}(\theta_e^{(k)})$ is the transformation matrix. To compute the updated stress field, only the $\mathbf{D}_e^{(k)}$ of elements with non-zero material density is calculated and sent to finite element analysis (FEA) software. For elements with zero material density in a given CFRP layer, no material is assigned in that layer, indicating that the fiber path does not fully cover the shell surface at that location.

3. One-stroke toolpath and printing trajectory generation for multi-axis printing

To fabricate the multilayer composite shell with optimized fiber paths, the fibers extracted in Section 2.3.3 are post-processed and transformed into manufacturable printing trajectories. The procedure is illustrated in Fig. 7. Considering the manufacturing constraints, the fiber segments are reordered and connected to a one-stroke toolpath in Section 3.1. Then, an inverse bijective mapping is applied to the 2D toolpath. With velocity planning, material extrusion planning, and robot kinematics, the toolpath is converted to time-domain printing trajectories in Section 3.2.

3.1. One-stroke fiber path generation

The CFRP-AM process differs significantly from conventional material extrusion methods due to the rigid connection between the continuous fiber, the nozzle, and the build platform. In cases where the printing trajectory is discontinuous, fiber cutting becomes necessary, which not only increases the risk of printing defects but also diminishes the overall reinforcement effectiveness of the continuous fiber. Therefore, it is essential to efficiently connect segmented fiber paths and

generate a continuous, one-stroke printing trajectory. This section presents a method for converting a given set of discrete fiber segments $\mathbf{c}$, defined on a 2D rectangle domain $\mathcal{P}$, into a printable toolpath suitable for CFRP-AM. The problem is subdivided into two key components: (1) the optimization of the connection order among discrete fiber segments, and (2) the design of connection paths between these segments.

Let fiber paths in k th layer be $\mathbf{c}^{(k)} = \{\mathbf{c}_1^{(k)}, \mathbf{c}_2^{(k)}, \dots, \mathbf{c}_{N_s}^{(k)}\}$, which is obtained from the fiber extraction algorithm discussed in Section 2.3.3. In a typical multilayer composite design, adjacent layers contain intersecting fiber paths to support both longitudinal and transverse loads, achieving balanced mechanical properties. Here, the two-layer optimized fiber paths guided by the maximum and minimum principal stress are incorporated to generate a one-stroke toolpath for printing. Let $\mathbf{c} = \{\mathbf{c}^k, \mathbf{c}^{k+1}\}$ denote a set of discrete fiber segments, and the total number of fiber segments is N_s . Each segment $\mathbf{c}_i$ is defined by two endpoints $\mathbf{p}_{2i-1}$ and $\mathbf{p}_{2i} \in \mathbb{R}^2$, and an associated direction vector pair $\mathbf{d}_{2i-1}, \mathbf{d}_{2i} \in \mathbb{R}^2$ corresponding to the start and end directions. All endpoints lie along the boundary $\mathcal{B}$ of the rectangular domain $\mathcal{P}$. The goal is to determine an optimized connection sequence that minimizes the total connection distance between segments while satisfying geometric and manufacturing constraints. This is the Travelling Salesman Problem (TSP), a classical optimization problem which is solved by a genetic algorithm (GA).

The algorithm begins by selecting the first segment $\mathbf{c}_i$ and initializing an ordered set $\mathbf{P}_s \in \mathbb{R}^{2 \times 2n}$ with the segments' start and end points, as well as their associated direction set $\mathbf{d}_s \in \mathbb{R}^{2 \times 2n}$. For manufacturing convenience, the endpoint $\mathbf{p}_i$ closest to the origin $[0, 0]^\top$ is selected as the initial point $\mathbf{p}_1$. The index i determines the connection direction of each segment by a list $I_d \in \mathbb{Z}^n$ and is known as

$$I_d = \begin{cases} 1 & i \in 2\mathbb{Z} + 1 \\ -1 & i \in 2\mathbb{Z} \end{cases}, \quad (36)$$

when I_d is positive, the fiber segment keeps its original direction, which goes from the start point to the end points; when I_d is negative, the sequence of $\mathbf{c}_i$ becomes inverse. Then, the last point of the connected path becomes

$$\mathbf{p}_c = \begin{cases} \mathbf{p}_s\{i+1\} & i \in 2\mathbb{Z} + 1 \\ \mathbf{p}_s\{i-1\} & i \in 2\mathbb{Z} \end{cases}, \quad (37)$$

and its direction is changed to

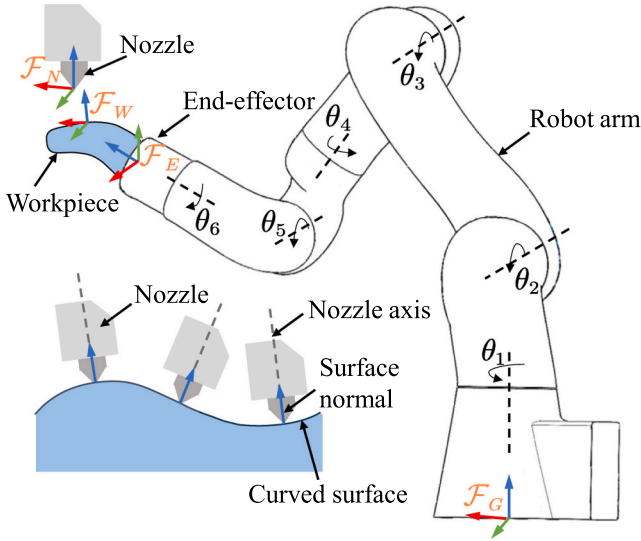


Fig. 9. Schematics of a robot-based multi-axis CFRP-AM system and fiber deposition direction.

$$\mathbf{d}_c = \begin{cases} \mathbf{d}_s\{i+1\} & i \in 2\mathbb{Z}+1 \\ -\mathbf{d}_s\{i-1\} & i \in 2\mathbb{Z} \end{cases} \quad (38)$$

At each iteration, the algorithm selects the unconnected endpoints $\mathbf{P}_u \in \mathbf{P}_s \setminus \mathbf{V}$, where $\mathbf{V}$ is the set of already visited endpoints, such that the distance from the last endpoint $\mathbf{p}_c$ of the current path to the next connected point $\mathbf{p}_n$ is minimized. Since all extracted fiber segment endpoints lie on the boundary $\mathcal{B}$ of the rectangular plane $\mathcal{P}$, naively connecting them results in zigzag trajectories with sharp turning angles. Due to the inherent stiffness of the continuous fiber, excessive turning angles may lead to defects such as fiber breakage or misalignment. Therefore, two constraints are imposed: a minimum allowable turning length $d_{\min}$, and a maximum allowable turning angle $\alpha_{\min}$ when the connection distance is less than d_t , as shown in Fig. 7. Then, the next promising endpoint $\mathbf{p}_n$ to connect to is obtained by

$$\begin{aligned} \mathbf{p}_n &= \mathbf{P}_u\{j\} = \operatorname{argmin}(\|\mathbf{p}_c - \mathbf{P}_u\|) \\ \text{s.t. } &\begin{cases} \|\mathbf{p}_n - \mathbf{p}_c\| \geq d_{\min} \\ \alpha_t \leq \alpha_{\min}, \text{ when } \|\mathbf{p}_n - \mathbf{p}_c\| \leq d_t \end{cases} \end{aligned} \quad (39)$$

where α_t is the turning angle between adjacent segments, and is calculated by

$$\alpha_t = \arccos\left(\frac{\mathbf{d}_c \cdot \mathbf{d}_n}{\|\mathbf{d}_c\| \|\mathbf{d}_n\|}\right), \quad (40)$$

where $\mathbf{d}_n$ is the direction vector at points $\mathbf{p}_n$. Fig. 8 depicts a set of fiber segments extracted from an optimized scalar function, and fiber segments are connected with the above-mentioned constraints.

Once a complete traversal sequence is constructed, the connection paths between consecutive points are generated with an additional boundary extension. While fiber intersections within a single layer can enhance structural strength, they also increase the risk of print failure due to material accumulation. Directly connecting endpoints located at boundary $\mathcal{E}$ with straight lines will result in excessive overlaps. To mitigate this, an extended region $\mathcal{C}$ is introduced along all four edges of the plane. Offsets of connected lines at the boundary are introduced into this region and then interconnected, thereby reducing the likelihood of overlaps within the main print area. Specifically, each endpoint $\mathbf{p}_i$ is extended outward of its located edge by a length $L_{o,i}$ given by

$$L_{o,i} = \frac{1}{2}L_{o,\min}(1 + \delta_i), \quad (41)$$

where $L_{o,\min}$ is the minimum extension length, and δ is a variable to introduce variability and avoid repeated intersections of the toolpath, which is defined by

$$\delta_i = \frac{i}{N_s} \in [0, 1], \quad (42)$$

ensuring the extended length is in range $[L_{o,\min}, 2L_{o,\min}]$. For the two endpoints on different edges, the shortest distance along the boundary direction is generated to connect them. The connection sequence of a generated one-stroke path is shown in Fig. 8(c). When the resulting 2D trajectory is mapped back onto the 3D surface, this extended region creates an overlap zone at the developing line, which facilitates smooth transitions between fiber segments and enhances inter-layer bonding strength. For multi-layer one-stroke toolpath generation, the endpoint of the current toolpath is set as the starting point of the next layer's optimization, with the layer height raised by layer thickness t .

Defining the one-stroke fiber path in the 2D domain as $\hat{\mathbf{P}}(l) : l \in [0, 1] \rightarrow \mathbb{R}^2$ and l is the percentage of the total length. The $\hat{\mathbf{P}}(l)$ can be conformally mapped to $\mathbf{P}(l) \in \mathbb{R}^3$ by

$$\mathbf{P} = f^{-1}(\hat{\mathbf{P}}) = [\mathbf{P}_x \quad \mathbf{P}_y \quad \mathbf{P}_z]^\top. \quad (43)$$

The curve length of the 3D fiber path is given by

$$s(l) = \int_0^l |\mathbf{dP}| = \int_0^l \left| \frac{\partial \mathbf{P}(l)}{\partial l} \right| dl. \quad (44)$$

Note that the curve length $s(l)$ monotonically increases based on the definitions, which indicates l and s formulate a one-to-one projection. Thus, the curve length parameterized fiber path is obtained and expressed by $\mathbf{P}(s)$.

3.2. Multi-axis printing trajectory generation

Considering the multi-axis printing process shown in Fig. 9, the printing part with a complex surface is connected and fixed to the end-effector. The fiber path $\mathbf{P}(s)$ is transformed into the fiber depositing position, also called the workpiece position, by

$${}^E\mathbf{P}_w = \mathbf{P} + \mathbf{P}_0, \quad (45)$$

where $\mathbf{P}_0$ is the initial printing position from the end-effector, and the superscript E represents ${}^E\mathbf{P}_w$ is defined in the end-effector reference frame $\mathcal{F}_E$. To guarantee sufficient contact between deposited fiber and the surface, and avoid collision between the nozzle and complex surface, the local surface's normal direction should align with the nozzle axis orientation, as shown in Fig. 9. The position ${}^G\mathbf{P}_e$ and orientation ${}^G\mathbf{O}_e$ of the robot end-effector could be derived with matrix transformation, and the details are discussed in [42].

Assuming the depositing velocity as V_p , the curve length with respect to time t is obtained by

$$s(t) = V_p t. \quad (46)$$

The time-domain position ${}^G\mathbf{P}_e(t)$ and orientation ${}^G\mathbf{O}_e(t)$ of the robot end-effector in each time step t_s are thus derived. The ${}^G\mathbf{P}_e$ and ${}^G\mathbf{O}_e$ are used for inverse kinematic (IK) calculation to obtain the joint angles of the robot arm for execution. For any configuration of the 6-axis robot arm, the joint angles can be calculated by the mathematical method based on the robot arm's structure, and the solution is expressed as

$$\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^\top. \quad (47)$$

Generally, the calculated results from the IK solver consist of multiple solutions. To ensure the robot arm executes a continuous and smooth motion, the best solution is selected by minimizing the distance between

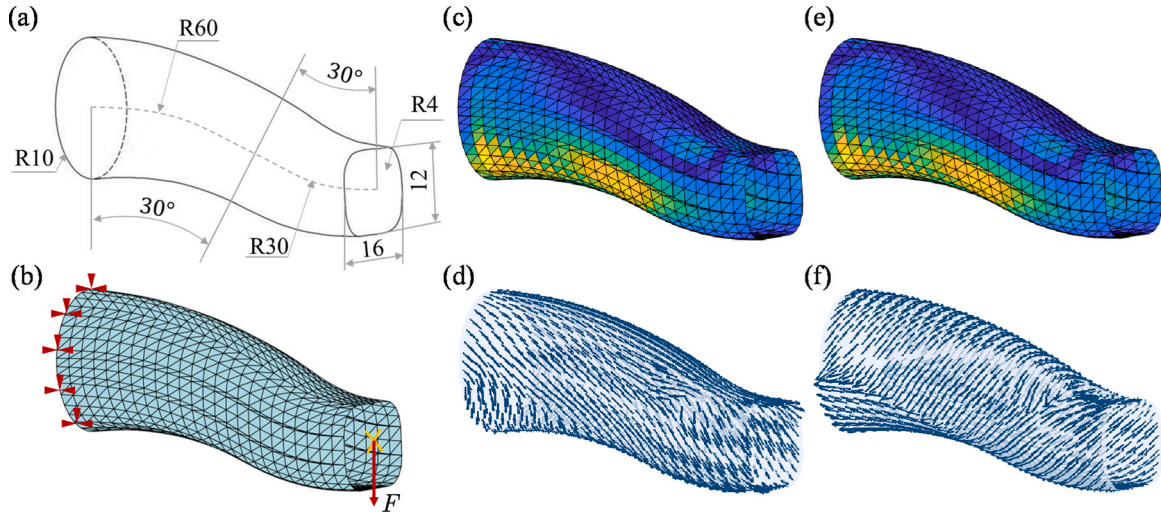


Fig. 10. (a) 3D model of curved tube with its (b) loading condition and corresponding (c), (d) maximum and (e), (f) minimum principal stress value and orientation.

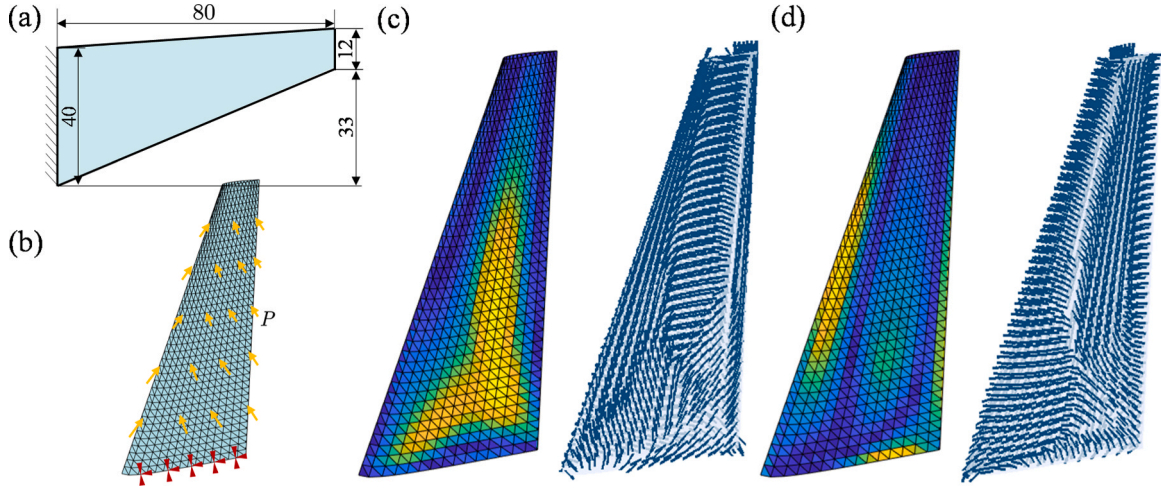


Fig. 11. (a) 2D and (b) 3D model of an airfoil with its loading condition and corresponding (c) maximum and (d) minimum principal stress value and orientation.

the current posture and the next posture, shown as

$$\Theta = \operatorname{argmin} \left(\sum_{i=1}^6 w_i \|\theta_i - \theta_i^*\| \right), \theta_i \in \hat{\Theta}, \quad (48)$$

where θ_i^* represents the current joint angles of the robot arm, w_i represents the weight and $\hat{\Theta}$ represents the joint limits. The thermoplastic matrix material extrusion lengths are assumed to be proportional to the curve length and given by

$$L_m = \beta_m s, \quad (49)$$

where β_m represents the extrusion rate ratio of the matrix.

4. Simulation and experiment verification

This section designs two case studies, a curved tube under bending and an airfoil under face pressure, to verify the proposed evolving stress-guided (ESG) optimization method established in Section 2 and the one-stroke toolpath and printing trajectory generation approach proposed in Section 3.

The curved tube, as illustrated in Fig. 10 (a) and (b), serves as a structural connector between two offset cross-sections of differing shapes. The tube functions as an arm link, smoothly transitioning

between the two ends over a specified spatial offset. Boundary conditions are defined such that the left end of the tube is fully fixed, while the right end is modeled as a rigid cross-section subjected to a downward force of 500 N applied at its centroid. The performance of the optimized design is assessed through both numerical simulations and physical experiments. Structural stiffness is evaluated as the ratio of the applied bending force to the resulting displacement at the point of loading. The airfoil structure, as shown in Fig. 11 (a) and (b), adopts profile NACA-4420 as its cross-section with a taper ratio of 0.3 and a total length of 80 mm. The root section is fully fixed, and a uniform pressure load of 0.1 N/mm² is applied to the surface to represent aerodynamic loading under a simplified flight condition. The performance of the optimized airfoil design is assessed using numerical simulations, with structural compliance employed as the primary evaluation metric.

The ESG optimization method is implemented to generate fiber paths with enhanced structural stiffness. The optimization proceeds until the total material volume reaches the constraint $V_{\max}$. To demonstrate the superiority of the proposed ESG method, conventional CSO fiber paths are selected as a reference design pattern for comparison [3]. The constant spacing s_c is defined on the 2D domain, and orientation is set to be $\pm 45^\circ$, since the $\pm 45^\circ$ fiber pattern is widely used in CFRP shell structures due to its high symmetry and balanced mechanical performance under multi-directional loading conditions [43]. Additionally, a general stress-guided (SG) optimization method without considering the fiber

Table 2
Material parameters of fiber and matrix material.

Parameter	PLA [22]	CFRP
Diameter [mm]	1.75	1.6
Density [g/cm^3]	1.25	2.8
Elastic modulus E_1 [MPa]	2205.18	16322.17
Elastic modulus E_2 [MPa]	-	9930
Shear modulus G_{12} [MPa]	-	4138
Poisson ratio μ_{12}	0.36	0.2

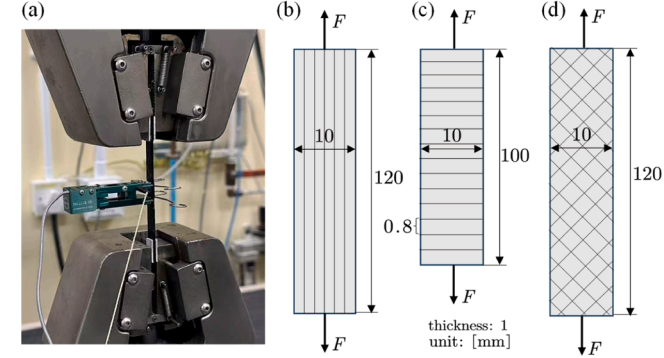


Fig. 12. (a) The experiment platforms of tensile test with (b) longitudinal, (c) transverse, and (d) shear specimens.

reinforcements [8,26] is selected for comparison. The constant spacing s_c and density factor K_d are adjusted to ensure the total material volumes of fiber patterns with different methods have a variance less than 5 %, to avoid the influence of material volume on mechanical properties.

4.1. Simulation and experiment platform

The proposed methodology is run on a desktop btained on a desktop PC with an Intel(R) Core(TM) i5–13500HX CPU+ 16 GB RAM. The FEA is conducted in the commercial software Abaqus. The model is structurally meshed into S3 triangular shell elements within a reference edge length of 2 mm, which remains the same during the optimization process. The composite layup module in Abaqus is employed to model the multilayered composite shell structure. For each layer (ply), the material properties, thickness, and fiber orientation are specified at the element level, which is accomplished by programmatically updating the material definitions in the Abaqus input (.inp) files at each optimization iteration, thereby ensuring that the evolving anisotropic behavior of the laminate is accurately captured in the FEA. The base layer is made of polylactic acid (PLA) material, and then continuous fiber-reinforced thermoplastic

(CFRP) layers are added to it. Based on the manufacturing constraints, the thickness for PLA and CFRP shells is set as 0.3 mm and 0.2 mm, respectively. The mechanical properties of PLA and CFRP materials are summarized in Table 2. To define the elastic mechanical properties for printed CFRP materials, the specimens are designed following the ISO 527–4 3 type standard as illustrated in Fig. 12. The fiber spacing is set to 0.8 mm, and the layer height is set to 0.2 mm to represent the case when the fiber volume fraction of the printed CFRP parts has the maximum manufacturable value. Under this condition, the material density of the CFRP material is one. Tensile tests were conducted on specimens, with an extensometer used to measure the force-displacement relationship within the gauge length, as shown in Fig. 12 (a). The specimens with 90° (Fig. 12(b)), 0° (Fig. 12(c)), and ±45° (Fig. 12(d)) fibers are used to measure the longitudinal, transverse, and shear properties, respectively. After applying the specific loading conditions to the shell structure, the FEA result with element information and 3D stress distribution is obtained and exported for further processing. Then, the FEA post-processing, fiber path optimization, and printing trajectory generation algorithms are implemented in MATLAB.

The manufacturing of multilayer shells with optimized fiber is achieved with a home-made robotic-based multi-axis CFRP-AM system, shown in Fig. 13. An industrial 6-axis industrial robot (Huashu Co603), a self-designed continuous fiber and thermoplastic co-extrusion system, are integrated and controlled with a Beckhoff TWINCAT real-time control system with EtherCAT communication and 1 kHz sampling rate. A print base with complex surfaces, which refers to the base PLA layer with 0.5 mm thickness, is printed by the commercial 3D printer (Creality K1 Max) using PLA filament and connected to the end-effector of the industrial robot before printing.

During the printing process, the extruder and load cell are fixed, and the industrial robot and print base move to execute the joint space trajectory Θ . The continuous fiber adopted is continuous dry basalt fiber, which is soft compared to fiber prepreg, enabling the printing of a steered path with higher accuracy. The fiber roll rotates at a constant speed to ensure the fiber is in a relaxed state before entering the extruder. The dry basalt fiber is threaded through the nozzle before

Table 3
Parameters of robotic-based multi-axis CFRP-AM platform.

Parameter	Values
Nozzle diameter	0.8 mm
Matrix material	PLA
Continuous fiber	Dry basalt fiber–0.4 K
Nozzle temperature	220 °C
Fiber printing velocity	3 mm/s
β_m	0.15

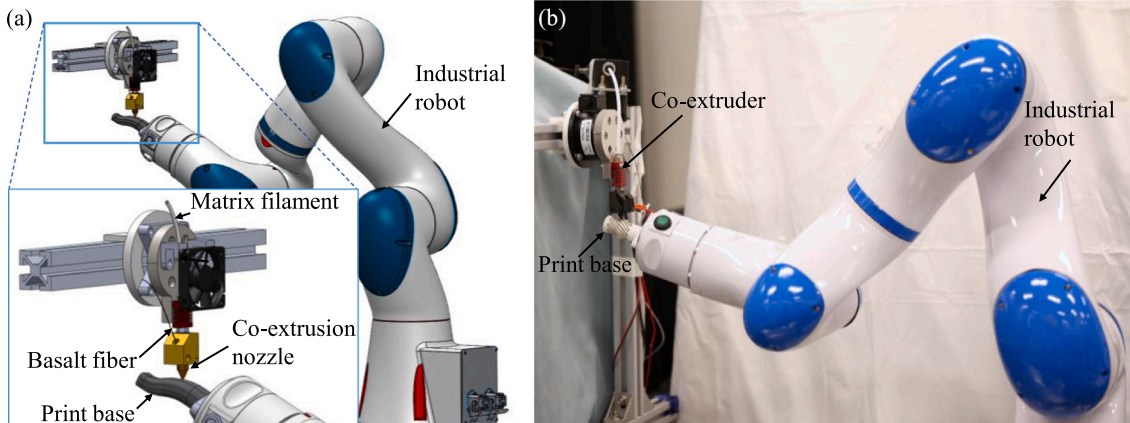


Fig. 13. (a) Schematics and (b) photo of multi-axis CFRP-AM experimental setup.

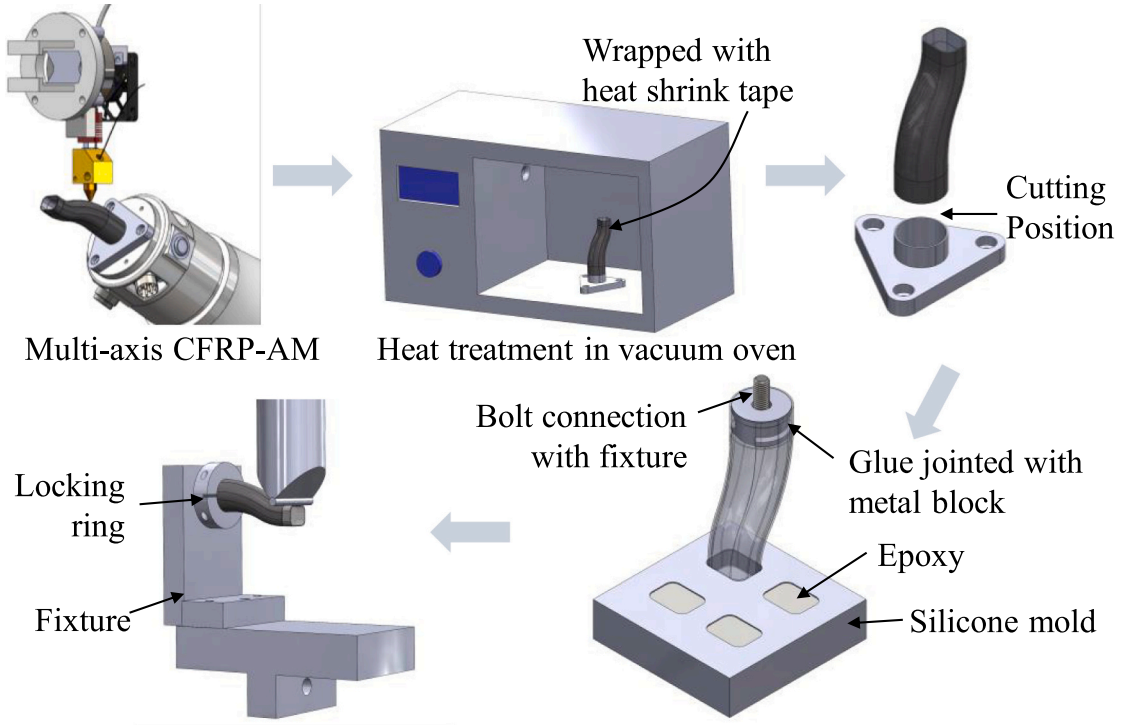


Fig. 14. Sample printing, post-processing, and preparation procedure for the bending test.

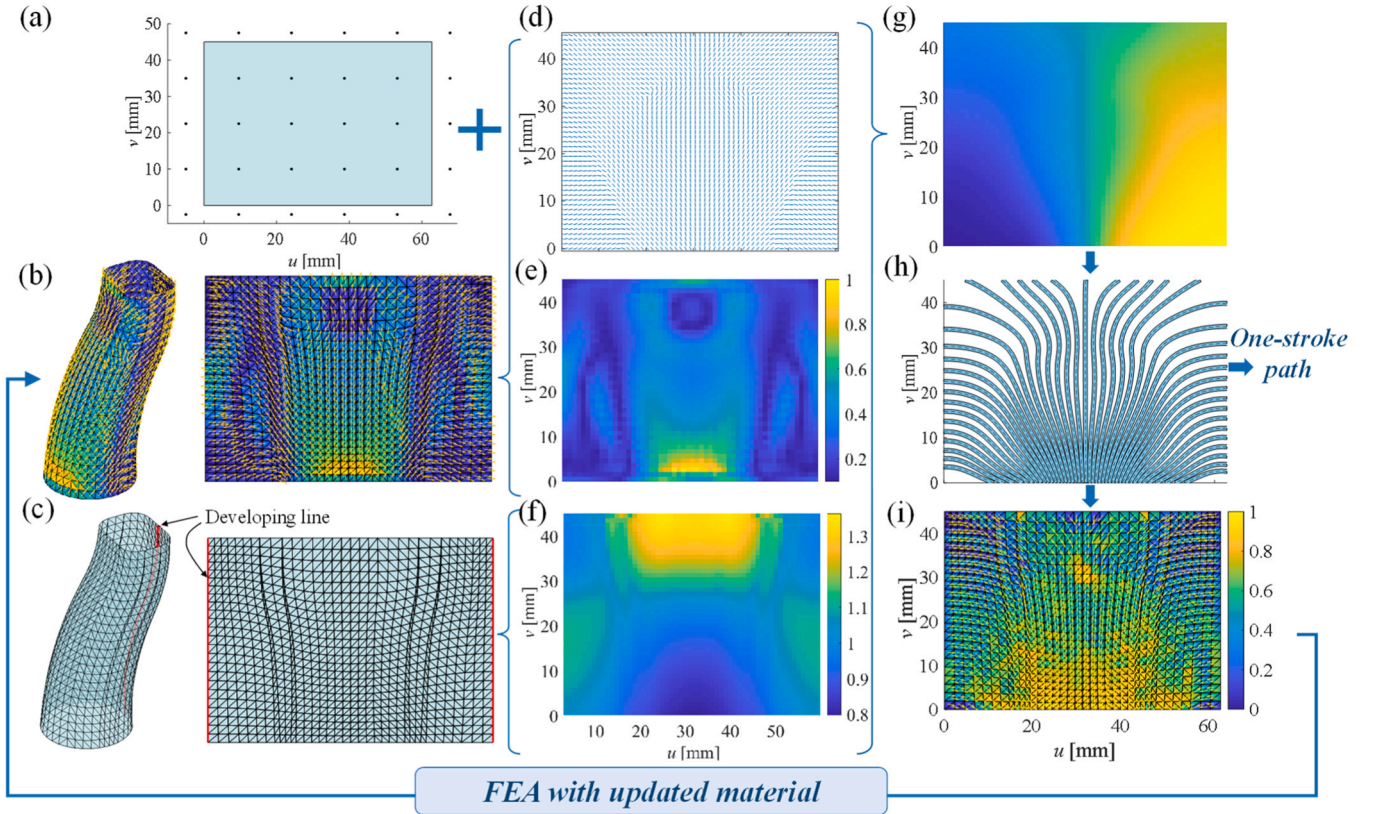


Fig. 15. Fiber path optimization procedure of a curved tube. (a) Scalar function $\phi(u,v)$ initialization with control points, (b) absolute maximum principal stress distribution, and (c) conformal mapping result of curved tube. (d) 2D mapped maximum principal stress direction, (e) reference CFRP material density, and (f) local length distortion map. (g) Optimization result of the scalar function, (h) extracted fiber segments, and (i) CFRP volume fraction and orientation evaluation.

Table 4

The parameters for iterative optimization and fiber path extraction for curved tube.

m_u, m_v	n_u, n_v	w_1, w_2, w_3, w_4	K_d	Δh [mm]	$\rho_{\min}, \rho_{\max}$	$V_{\min}$ [mm ³]
2, 2	6, 5	1, 1, 1000, 5	1	0.2	0, 1	2240

printing. During the printing process, the matrix filament is fed by a stepper motor with length L_m and melted in the heated chamber. The dry fiber is in-situ impregnated with melted resin, adheres to the print base, and drags the subsequent fiber out. The nozzle temperature is kept at 220 °C to melt the matrix PLA. The detailed CFRP-AM system parameters are listed in Table 3. The printed shell is postprocessed to be able to conduct a bending test, and the sample preparation procedure is illustrated in Fig. 14. The printed basalt fiber/PLA shell samples are wrapped with heat shrink tape and sent to a vacuum oven at 80 °C for 3 h to achieve sufficient fiber impregnation [44,45]. The part designed for connection with the end effector is cut off, leaving the left part with

composite layers. Multiple connections are introduced to fix the curved tube onto the fixture for the bending test. The left inner surface of the curved tube is glued joint with an aluminum alloy cylinder with a height $2L_{o,\min 1}$, which has a screw inserted in its center and forms a bolted connection with the fixture. The left outer surface of the curved tube is fixed by a locking ring. To prevent right cross-sectional deformation of the curved tube, epoxy is filled with $2L_{o,\min 2}$ depth. The bending test is performed on the MTS Sintech 10/D universal test machine (Fig. 25 (a)) with a speed of 5 mm/s.

4.2. Simulation and experiment results

4.2.1. Case study of a curved tube under bending

The FEA analysis was performed on the homogenized 0.3 mm PLA shell to obtain the initial stress field shown in Fig. 10 (c)-(f), which subsequently guided the optimization design of the first fiber layer. The maximum principal stress-guided fiber path optimization process is illustrated in Fig. 15 as a representative example. The maximum

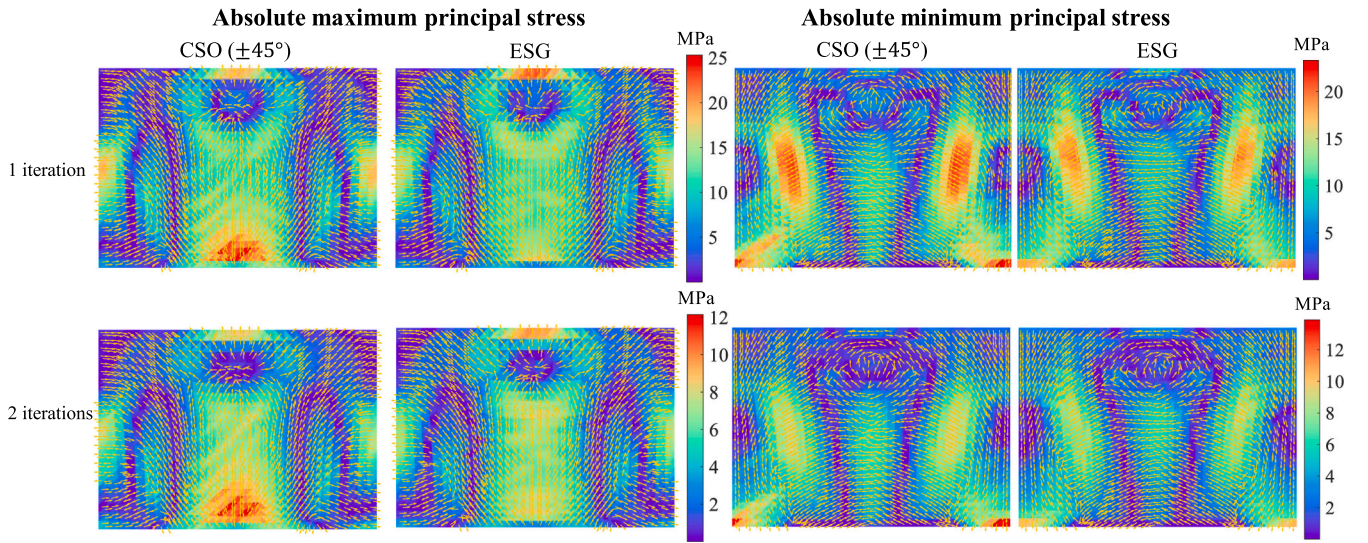


Fig. 16. Evolving stress field variations for multilayer composite shells with different fiber patterns.

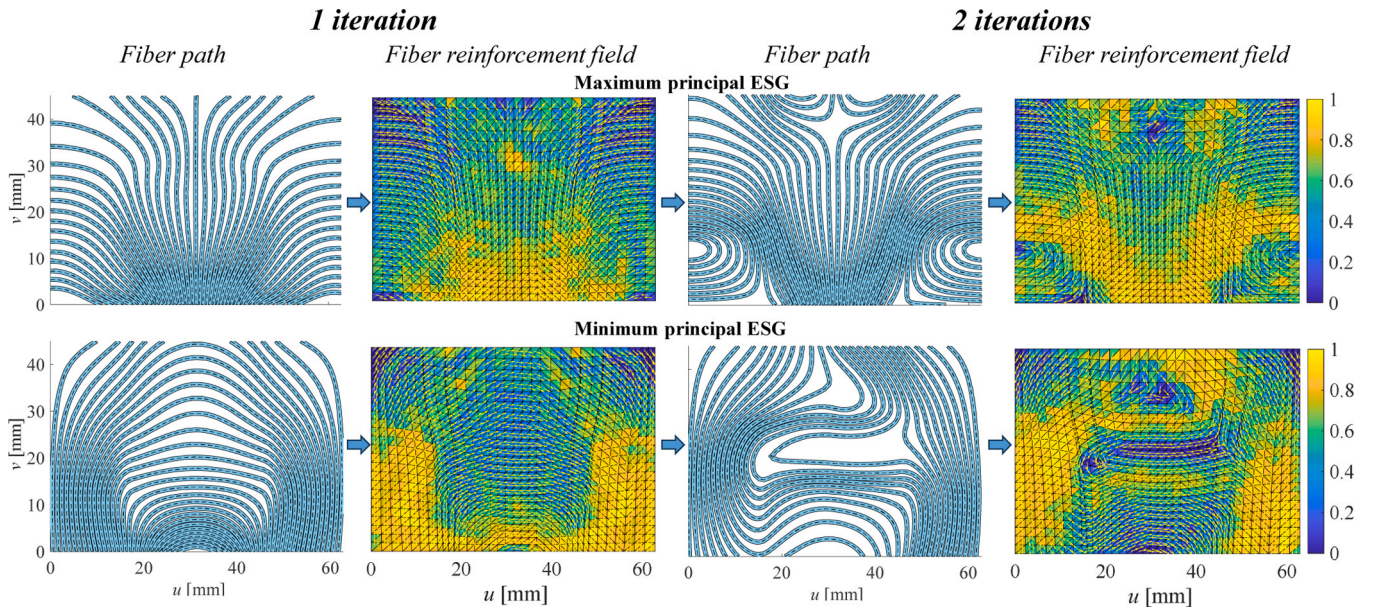


Fig. 17. Path and fiber reinforcement variations in the iterative optimization process.

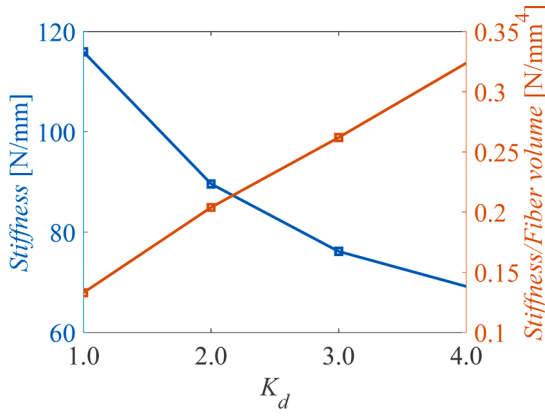


Fig. 18. Relationship between structural stiffness and stiffness/CFRP volume ratio with different density factors K_d .

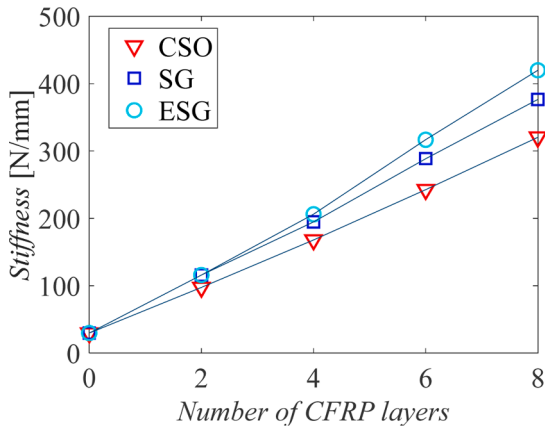


Fig. 19. Stiffness variation with increase of CFRP layer numbers for different fiber patterns with equal CFRP material volumes.

principal stress magnitude and orientation depicted in Fig. 15 (b) are mapped onto a 2D domain through the bijective mapping model established in Fig. 15 (c). The mapped element-based stress orientations are converted to directionless vectors, while the stress magnitudes are linearly scaled to the range $[\rho_{\min}, \rho_{\max}]$ to represent the reference CFRP volume fraction. These parameters are then interpolated to 2D parameterized functions $\tau^{(0)}$ and $\rho^{(0)}$, as presented in Fig. 12 (d) and (e), respectively. The corresponding length distortion map r_d is illustrated in Fig. 15 (f).

The parameters for iterative optimization are illustrated in Table 4. To represent the scalar function $\phi^{(1)}(u, v)$ for stress-guided fiber path optimization, a NURBS surface is constructed from a grid of control points P_{ij} . Control points are equidistantly distributed across the 2D domain with a 2.5 mm extension beyond each boundary. The NURBS surface parameters, specifically the orders m_u, m_v and the number of control points n_u, n_v , are selected to govern the degrees of freedom of the scalar function, as demonstrated in Fig. 15 (a). The weighting factors in Eq. (21) were determined through preliminary tests according to the representative magnitudes of each objective term. The first two terms,

which dominate the control of fiber path spacing and orientation, were both assigned unit weights ($w_1 = w_2 = 1.0$) to achieve a balanced trade-off between orientation fidelity and spacing uniformity. For the extremal-point penalty term, a relatively high weight ($w_3 = 1000.0$) was applied to suppress the formation of closed contours and thereby facilitate the generation of a continuous one-stroke toolpath. In the case of closed-shell structures, an additional boundary continuity constraint was introduced, with weight $w_4 = 5.0$, to promote smooth fiber alignment across boundary regions. The optimization problem is solved using MATLAB's built-in particle swarm optimization (PSO) algorithm over 300 iterations to obtain an initial solution. This is followed by a

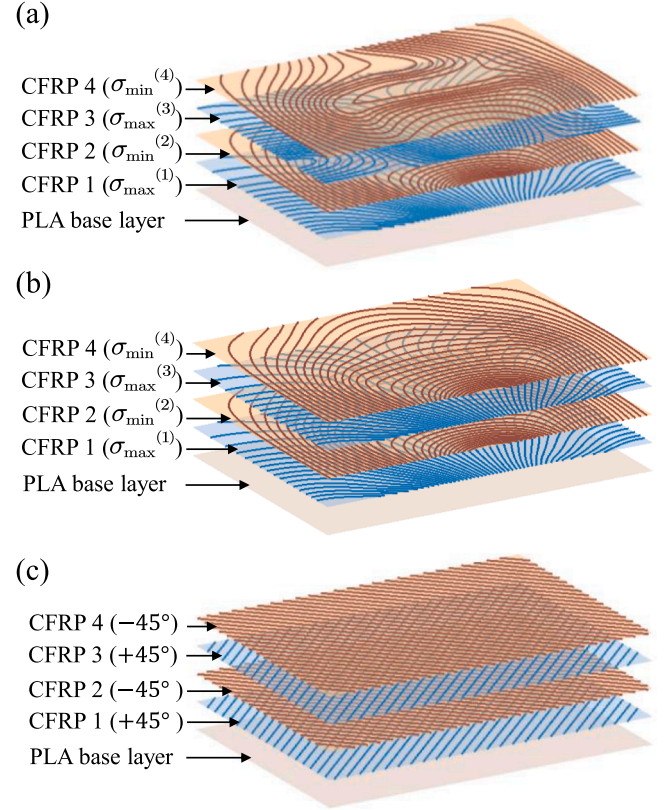


Fig. 20. Multi-layer stacking distribution of composite shells of curved tube with (a) ESG, (b) SG, and (c) CSO fiber path patterns.

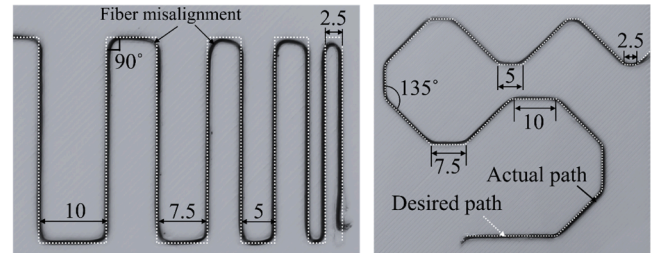


Fig. 21. Fiber deposition results with different turning angles and lengths.

Table 5
Stiffness variation with iterations of different fiber patterns with equal CFRP material volumes.

	CFRP layer number	CSO [3]	SG [8,26]	ESG (proposed)	Enhancement compared to CSO	Enhancement compared to SG
Stiffness [N/mm]	2	97.6	115.9	115.9	21.1 %	-
	4	168.0	121.4	194.8	21.3 %	5.9 %
	6	242.6	78.7	288.7	34.6 %	9.8 %
	8	320.51	57.0	47.9	48.9 %	19.0 %

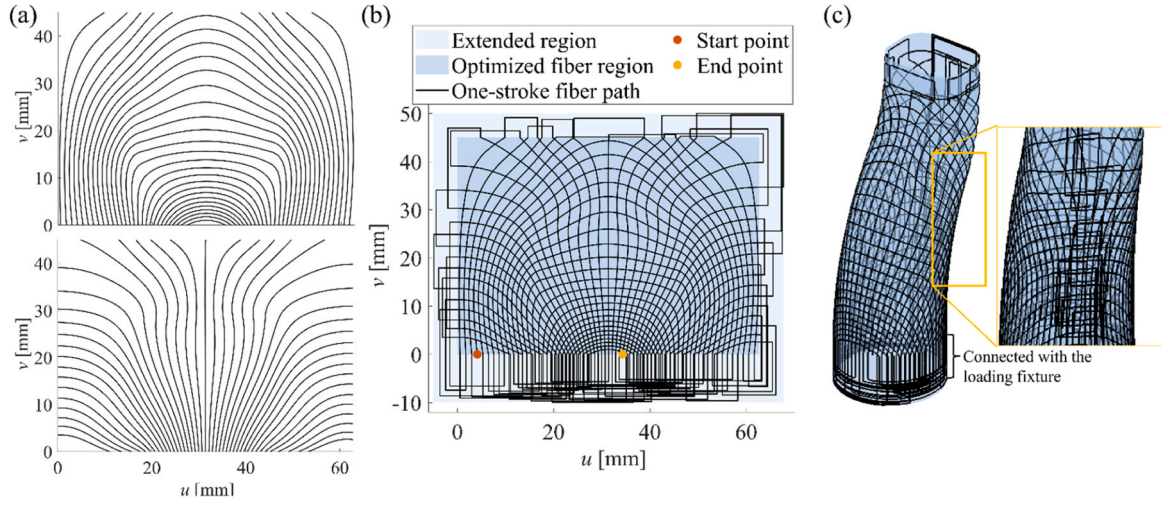


Fig. 22. One-stroke toolpath generation procedure. (a) Extracted fiber segments from the optimized maximum and minimum principal stress-guided scalar function, (b) one-stroke toolpath with extended region in the 2D domain, and (c) 3D surface.

Table 6

The parameters for one-stroke toolpath generation.

$d_{\min}$ [mm]	d_t [mm]	α_t [°]	$L_{o,\min1}$ [mm]	$L_{o,\min2}$ [mm]	K_d	s_c [mm]
5	10	90	5	2.5	1	1.75

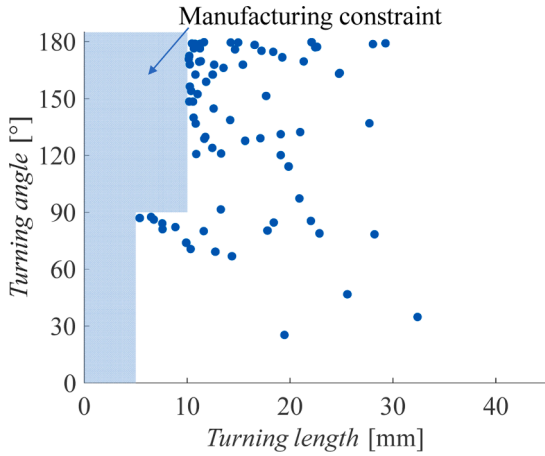


Fig. 23. Turning angle and length distribution of the one-stroke toolpath.

refinement step using the `fmincon` function to improve solution accuracy. The computational cost per optimization iteration using a mesh of 1728 elements is approximately 70.03 s.

Fig. 15 (g) presents the optimized scalar function, and the fiber path extraction algorithm is executed with a density factor K_d to generate fiber segments $\mathbf{c}^{(1)}$ in Fig. 15 (h). These fiber segments are subsequently transformed into a fiber distribution map containing CFRP volume fraction $\rho^{(1)}$ and orientation $\theta^{(1)}$, illustrated in Fig. 15 (i), which directly influences the elasticity matrix $\mathbf{D}^{(1)}$ of each element. Upon optimization of orthogonal fiber layers guided by the maximum and minimum principal stresses, they are integrated for FEA analysis with updated material properties and shell thickness, thereby serving as input for the subsequent optimization iteration. As the number of layers increases progressively, the stress field undergoes evolution throughout the optimization process. Fig. 16 illustrates the variation in principal stress distribution as the number of layers increases. After an iterative optimization cycle, with the addition of fiber paths, the resulting principal

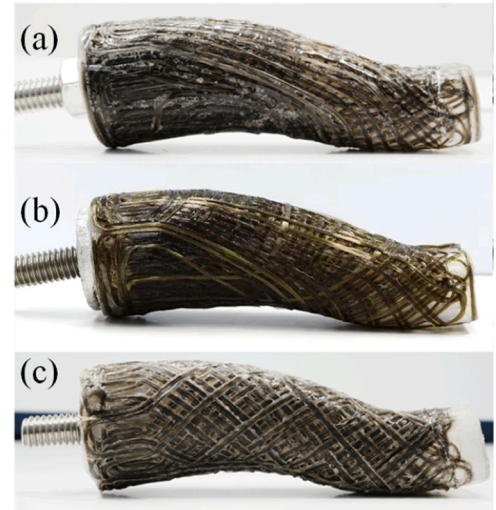


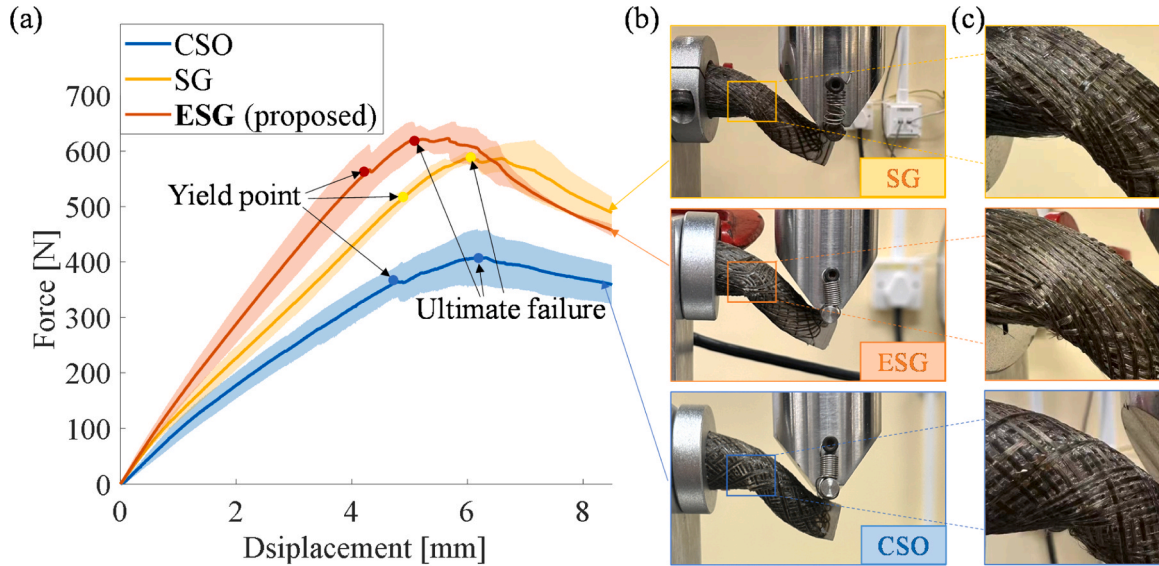
Fig. 24. Printed curved tube with (a) ESG, (b) SG, and (c) CSO fiber path patterns.

stress distribution patterns differ from the initial stress distribution shown in Fig. 10 (c)–(f). In contrast to CSO paths, the optimized ESG fiber paths are strategically aligned along principal stress directions, effectively mitigating stress concentration. Additionally, a notable reduction in maximum stress magnitude is observed in Fig. 16. These phenomena validate that fiber paths redistribute load transfer mechanisms through different fiber arrangements. An optimized stress-guided (SG) path results in improved structural performance and enhanced load-carrying capacity compared to CSO paths. With increasing iterations, the fiber paths in newly added layers are optimized based on the existing stress field, known as evolving stress-guided (ESG) fiber paths. Due to the variation of evolving stress fields, the optimized ESG paths and corresponding fiber reinforcement for each layer are distinct, as shown in Fig. 17.

To investigate the influence of CFRP volume fraction on structural stiffness, fiber segments are extracted using density factors K_d ranging from 1.0 to 4.0, obtained from two optimized scalar functions based on maximum and minimum principal stress fields. The stiffness characteristics of composite shells with varying fiber volume fractions are evaluated, with the results presented in Fig. 18. The analysis reveals that as the density factor K_d decreases—corresponding to an increase in fiber

Table 7Optimization results of curved tube with different fiber paths. The data is presented as mean value $\pm$ SD (n = 3).

		CSO [3]	SG [8,26]	ESG (proposed)	Enhancement compared to CSO	Enhancement compared to SG
Stiffness [N/mm]	Exp.	102.26 $\pm$ 23.89	126.14 $\pm$ 10.93	143.93 $\pm$ 7.81	57.70 %	19.59 %
	Sim.	168.01	194.83	206.10	22.67 %	5.78 %
Peak load [N]		413.43 $\pm$ 42.77	600.57 $\pm$ 20.39	653.75 $\pm$ 21.34	58.13 %	8.85 %
Fiber usage [mm]		6393.11	6469.8	6572.72	-2.81 %	-1.59 %

**Fig. 25.** (a) Force-displacement relationship comparison between three fiber path types, (b) corresponding failures with (c) fiber detachment and breakage region.

volume fraction—there is a notable enhancement in overall structural stiffness. To more effectively assess the contribution of fiber content to structural performance, we introduce the metric stiffness-to-fiber-volume ratio, which quantifies the efficiency of stiffness enhancement per unit of fiber material. The results indicate that although decreasing K_d (i.e., increasing fiber volume) leads to higher structural stiffness, the stiffness/fiber volume ratio declines accordingly. This suggests that the marginal stiffness gain per unit fiber becomes less efficient as fiber content increases, highlighting an important trade-off between material usage and mechanical performance. Nevertheless, designing fiber paths aligned with principal stress directions, in combination with increased fiber volume fraction, remains a critical strategy for approaching the upper limits of structural stiffness and strength enhancement [46]. Consequently, in subsequent simulations and experiments, $K_d = 1$ is adopted to achieve maximum fiber volume fraction within the constraints of layer number limitations, thereby optimizing mechanical performance enhancement.

To design a multi-layer composite shell achieving a minimized compliance with iteration optimization and also facilitate the comparison with experimental results, the stiffness in each iteration is calculated and evaluated. The structural stiffness of three composite shells with different fiber patterns at four iterations is illustrated in Fig. 19 and summarized in Table 5. As the number of iterations progressed, the quantity of CFRP layers increased correspondingly, resulting in a monotonic increase in structural stiffness. The ESG fiber paths demonstrated significantly increased stiffness values relative to CSO fiber configurations, with the magnitude of this enhancement exhibiting a positive correlation with layer count. Furthermore, the ESG paths yielded superior performance compared to SG paths, with performance improvements scaling proportionally with the number of CFRP layer increases. The ESG methodology incorporates the influence of fiber reinforcement effects by generating fiber paths based on the most recently computed stress field distribution. These results validate the

efficacy of the proposed iterative optimization framework that employs an evolving stress field to achieve enhanced structural performance.

To experimentally verify the proposed methodologies, composite shells with three different layer stacking strategies were printed using the proposed one-stroke toolpath generation approach. Fig. 20 illustrates the multi-layer stacking distribution. Each shell contains five layers: one 0.3 mm PLA base layer and four 0.2 mm CFRP layers. Fig. 20 (a) adopts ESG path patterns, where layers 3 and 4 are optimized from evolving stress, which integrates the fiber reinforcements from layers 1 and 2, resulting in different fiber arrangements from lower layers. Fig. 20 (b) adopts the SG method without considering fiber reinforcement contribution. These curvilinear fiber paths are extracted from an optimized scalar function with the density factor K_d . Fig. 20 (c) selects $\pm 45^\circ$ as fiber orientation, and s_c as constant spacing defined in the 2D domain to achieve an approximate fiber usage to curvilinear fiber groups.

To enable multi-axis CFRP-AM, the separated fiber segments are ordered and connected to a one-stroke toolpath with the method proposed in Section 3.1. To ensure the geometric accuracy of the deposited fiber paths, three key manufacturing constraints were established based on empirical observations: a minimum allowable turning length $d_{\min}$, and a maximum allowable turning angles α_t applicable within the turning length threshold d_t . These constraints were determined through a series of controlled deposition experiments, in which single fiber bundles were printed under varying turning angles and turning lengths. As illustrated in Fig. 21, fiber deposition at a turning angle of 180° exhibited acceptable alignment only when the turning length exceeded 10 mm. For 90° turns, deposition quality remained satisfactory for turning lengths greater than 5 mm. When these thresholds were not met, notable fiber misalignment and inward deviation were observed, indicating a loss of deposition fidelity. These experimental findings were used to define the manufacturability constraints integrated into the toolpath optimization process to ensure reliable fiber deposition. To

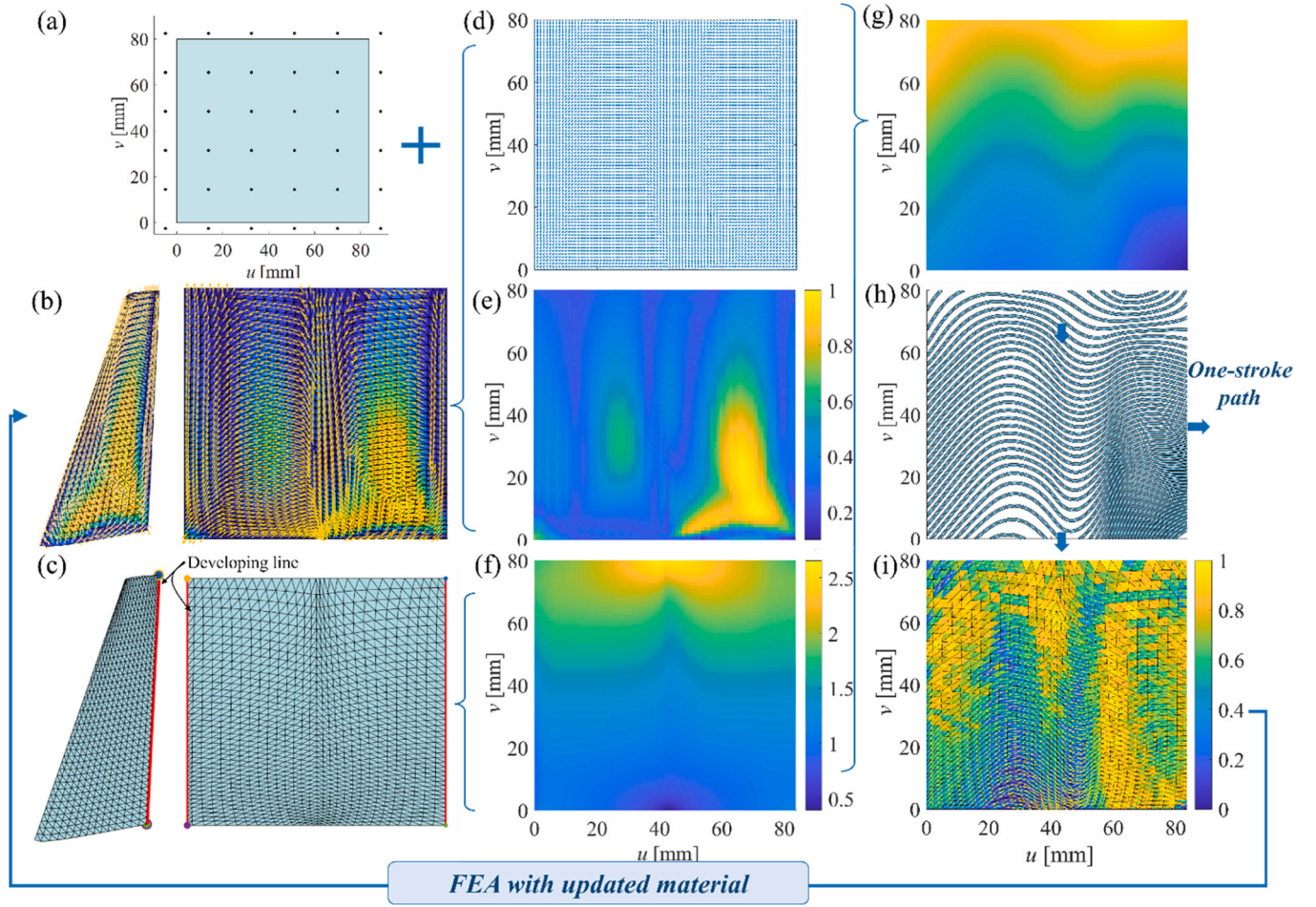


Fig. 26. Fiber path optimization procedure of an airfoil. (a) Scalar function initialization with control points, (b) absolute maximum principal stress distribution, and (c) conformal mapping result of curved tube. (d) 2D mapped maximum principal stress direction, (e) reference CFRP volume fraction, and (f) local length distortion map. (g) Optimization result of the scalar function, (h) extracted fiber segments, and (i) CFRP volume fraction and orientation evaluation.

Table 8

The parameters for iterative optimization and fiber path extraction for airfoil shell.

m_u, m_v	n_u, n_v	w_1, w_2, w_3, w_4	K_d	Δh [mm]	$\rho_{\min}, \rho_{\max}$	$V_{\max}$ [mm ³]	s_c [mm]
2, 2	6	1, 1, 1000, 0	1	0.2	0, 1	1620	2.2

make the printed curved tube conveniently installed with other connectors and facilitate further bending tests, the minimum extension length is set to be $L_{o, \min 1}$ with a larger value for the bottom boundary and $L_{o, \min 2}$ for other boundaries of the 2D domain. The genetic algorithm is selected to calculate the optimized one-stroke path as shown in Fig. 22 (b) and mapped to 3D paths shown in Fig. 22 (c). The regions with relatively large turning angles are located in the extended region, which is further connected with the loading fixture. The parameters for fiber path generation are listed in Table 6. To quantitatively assess the efficiency and manufacturability of the proposed one-stroke toolpath planning strategy, the distribution of turning angles and turning lengths for all connected fiber segments is analyzed and presented in Fig. 23. The results demonstrate that all turning lengths exceed the minimum allowable turning length of $d_{\min}$, thereby ensuring the feasibility of continuous fiber deposition without inducing excessive curvature. Moreover, for shorter connection segments (i.e., lengths less than d_t), the turning angles remain within the allowable range α_t , satisfying the turning constraints imposed by nozzle kinematics and fiber flexibility.

The overall distribution reveals that a majority of connections involve short segment lengths and moderate turning angles, reflecting an optimized and smooth toolpath with minimal travel. These findings confirm that the generated toolpaths are not only geometrically continuous but also kinematically feasible, supporting reliable implementation in CFRP-AM.

Following the sample preparation protocol outlined in Fig. 14, printed curved tubes incorporating optimized fiber layers were fabricated as depicted in Fig. 24. All specimens were fabricated under identical printing parameters, previously optimized and validated in [42], to ensure consistent fiber deposition quality across all fiber path strategies. To ensure the robustness of our experimental validation, the mechanical tests were conducted with three replicate specimens for each configuration. The mechanical testing results are summarized in Table 7, with corresponding force-displacement relationships and associated uncertainties presented in Fig. 25 (a). The solid curve represents the mean values across three independent measurements, with the shaded region indicating one standard deviation. The difference in fiber usage for different fiber patterns is less than 5 %. Tests revealed significant performance improvements: the ESG configuration demonstrated 57.70 % and 58.13 % enhancements in structural stiffness and peak load capacity, respectively, compared to conventional CSO fiber arrangements. Furthermore, when compared to the SG methodology, the proposed ESG approach achieved a 19.59 % and 8.85 % improvement in structural stiffness and peak load, respectively. The three repetitive experiments demonstrated the consistency and validity of the experiments.

The experimental stiffness enhancement is consistent with the

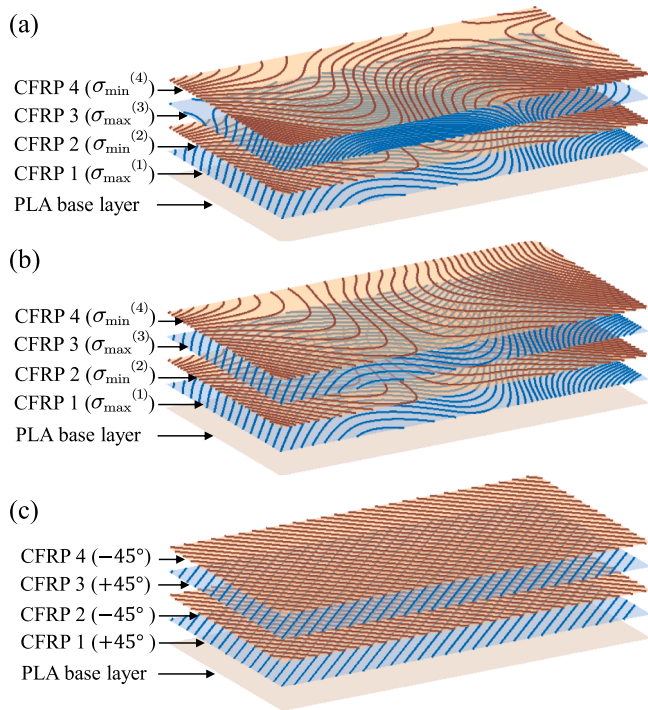


Fig. 27. Multi-layer stacking distribution of composite airfoil shell with (a) ESG, (b) SG, and (c) CSO fiber path patterns.

simulation results. However, the experimental stiffness enhancement is much larger than the predicted value from the simulation. Besides, the experimentally measured stiffness values are all lower than the simulated predictions across three methods; this discrepancy can be attributed to several factors. First, the simulations employ idealized material models assuming linear elastic and homogeneous orthotropic behavior, whereas the actual CFRP components exhibit variability due to resin-rich areas, fiber waviness, and voids. Second, manufacturing imperfections such as fiber misalignment, incomplete consolidation, and interfacial defects are inherent in the 3D-printed specimens, particularly in regions with dense fiber intersections, which may lead to friction-induced fiber abrasion. Finally, deviations in boundary conditions—such as fixture compliance and load eccentricity in the experimental setup—introduce additional deformations not accounted for in the numerical model. Despite these differences, the consistent ranking of stiffness across methods in both simulation and experiment supports the robustness and effectiveness of the proposed ESG optimization strategy.

The enhancement in fracture strength, as evidenced by the increased peak load, can be attributed to the improved load-bearing capacity of the embedded fibers aligned along the shell surface. These strategically oriented fibers contribute to a more efficient redistribution of the stress field, thereby mitigating stress concentrations and delaying the initiation of localized failure mechanisms. It should also be noticed that the ESG specimen exhibits an earlier onset of peak load, despite achieving a higher maximum load capacity, as illustrated in Fig. 25 (a). This behavior is primarily attributed to the transverse (bending) loading applied to the curved composite tube, which induces significant through-thickness shear stresses. While the ESG method effectively enhances in-plane load-bearing capacity by aligning fiber paths with the principal stress directions on the shell surface, it does not reinforce the

through-thickness direction. During the CFRP-AM process, insufficient in-situ resin impregnation often leads to the formation of voids concentrated within the core of the fiber bundle, resulting in unimpregnated zones. These voids are well-known to act as initiation sites for interlaminar cracking under mechanical loading [47]. Furthermore, the continuous fibers are bonded solely by the matrix, which results in relatively weak fiber–matrix interfacial adhesion. This weak interface facilitates fiber pull-out during failure, thereby undermining load transfer efficiency and diminishing the overall mechanical integrity of the structure [48]. Consequently, weak interlaminar bonding between printed CFRP layers emerges as a critical failure mechanism under high loading conditions. Once the peak load is reached, interlayer delamination is initiated due to the limited through-thickness shear resistance, leading to premature structural failure. This failure mode is further supported by the experimental observations in Fig. 25 (c), where fiber bundle detachment and localized breakage are evident indicators of interfacial debonding and delamination propagation.

The failure mechanisms exhibited by the three specimens with distinct fiber patterns were comparable, as illustrated in Fig. 25 (c). Under large deformation conditions, buckling occurred at the posterior center region of the curved tube, accompanied by the fracture of multiple fiber filaments. The stress-guided fiber paths, oriented along principal stress directions, effectively sustained elevated stress levels, thereby contributing to enhanced overall mechanical performance. However, a critical limitation was identified in the interlayer bonding characteristics of these multi-layer composite shells. It could be observed that a propensity for fiber filament delamination from the shell surface, attributed to interlayer shear forces. This phenomenon suggests that future investigations should focus on multi-layer fiber path optimization with explicit consideration of interlayer bonding mechanisms to achieve superior performance enhancement.

4.2.2. Case study of an airfoil under pressure

The FEA analysis was conducted on the homogenized shell model to obtain the initial stress distribution, as shown in Fig. 11 (c) and (d). This stress field served as the basis for optimizing the fiber orientation in the first ply. A representative case of the maximum principal stress-guided fiber path optimization is illustrated in Fig. 26. The magnitudes and orientations of the maximum principal stresses, presented in Fig. 26 (b), are mapped onto a 2D domain using the bijective mapping framework described in Fig. 26 (c). Within this mapped domain, the element-wise stress orientations are transformed into directionless vectors, while the corresponding stress magnitudes are linearly normalized to a specified range $[\rho_{\min}, \rho_{\max}]$, which reflects the reference CFRP volume fraction. These quantities are subsequently interpolated into continuous 2D parametric functions $\tau^{(0)}$ and $\rho^{(0)}$, as presented in Fig. 26 (d) and (e), respectively. In addition, the associated length distortion map r_d , which captures geometric deformation effects, is illustrated in Fig. 26 (f).

The parameters used for the iterative optimization process are presented in Table 8. To represent the scalar function for stress-guided fiber path optimization $\phi^{(1)}(u, v)$, a NURBS surface is constructed using the parameters m_u, m_v, n_u, n_v , as is illustrated in Fig. 26 (a). The weighting factors $w_1 \sim w_4$ applied are consistent with those employed in the curved tube case study, with the exception that w_4 is set to zero. This adjustment is made because the developing line in the mapping is selected as the trailing edge of the airfoil, where maintaining fiber path smoothness at the boundary is of lesser importance. Due to the increased geometric complexity and larger design domain, the optimization for the

Table 9

Compliance variation with iterations of different fiber patterns with equal CFRP material volume.

	CFRP layer number	CSO [3]	SG [8,26]	ESG (proposed)	Enhancement compared to CSO	Enhancement compared to SG
Compliance [J]	2	71,118.5	67,258.1	67,258.1	5.7 %	-
	4	39,722.3	36,872.9	32,949.1	20.6 %	11.9 %

airfoil is performed over 600 PSO iterations, followed by refinement with *fmincon*. The mesh for this case consists of 2400 elements, and the average computational cost per optimization iteration is approximately 338.58 s. The increased runtime compared to the tube case is primarily attributed to the larger optimization domain, which requires a greater number of iterations and higher computational effort to achieve convergence. The entire optimization process is carried out over two iterations, with structural compliance evaluated for composite shells utilizing ESG, SG, and CSO fiber patterns. To ensure a fair comparison, the total CFRP material volume across the three groups is maintained within a 5 % margin by adjusting the density factor K_d and constant spacing s_c , thereby isolating the influence of fiber path design on mechanical performance.

Fig. 27 illustrates the multi-layer stacking distribution of the composite airfoil shell with different fiber patterns. Each shell contains five layers: one 0.3 mm PLA base layer and four 0.2 mm CFRP layers. The structural compliance of the three composite shells after two optimization iterations is summarized in Table 9. After the first iteration, where two additional CFRP layers are introduced, the shell with SG fiber paths demonstrates lower compliance compared to the unoptimized CSO group. In the second iteration, the ESG configuration, which incorporates updated stress fields informed by the fiber reinforcements, exhibits a further reduction in compliance, outperforming the SG group. These results underscore the effectiveness of the proposed stress-guided optimization method in enhancing the structural performance of composite shells.

5. Conclusion

This study presents a comprehensive computational framework for the design and manufacturing of mechanically optimized, multi-layered laminated composite shells, with a specific focus on continuous fiber-reinforced polymer (CFRP) structures fabricated via 3D printing. The proposed framework addresses the geometric complexity of shell structures, the multilayer stress-guided and fiber-reinforcement feedback optimization, and the practical constraints of extrusion-based additive manufacturing (AM). To ensure manufacturability using extrusion-based CFRP-AM, a one-stroke toolpath planning algorithm is introduced. This algorithm addresses common manufacturing constraints, including maximum allowable turning angle and minimum turning length, while maintaining fiber continuity.

Two case studies, a curved composite tube under bending and an airfoil shell under face pressure, were designed to validate the proposed framework. Simulation and experimental results from multi-axis CFRP-AM confirmed the efficacy of the proposed evolving stress-guided (ESG) optimization method. Compared to conventional constant-spacing and orientation (CSO) fiber patterns as well as existing stress-guided (SG) methods, the ESG-optimized structures demonstrated significant improvements in structural stiffness and peak load-bearing capacity. Moreover, the performance gains were found to scale proportionally with the number of CFRP layers, highlighting the benefit of incorporating updated stress fields and reinforcement effects into the iterative optimization process. Experimental measurements of stiffness exhibited good agreement with simulation results, with minor discrepancies attributed to modeling simplifications and unavoidable manufacturing imperfections.

While the current framework offers valuable contributions, it also presents several areas for improvement. First, the parametric mapping method is based on quasi-conformal mapping, which restricts its applicability to genus-zero surfaces (i.e., surfaces without holes). Extending the method to handle multiply-connected domains remains a challenge. Second, the fiber path optimization strategy focuses exclusively on in-plane stress distributions, neglecting through-thickness reinforcement and interlaminar strength, which are critical factors in laminated composite structures and particularly for delamination resistance. Third, the compliance-based optimization targets elastic behavior and does not

explicitly consider failure criteria such as strength, delamination, or progressive damage. Furthermore, the mechanical loading conditions considered in this study are limited to static cases. Future research will focus on the following directions: incorporating through-thickness reinforcement strategies and interlayer bonding considerations to address the critical issue of delamination in laminated composites; more comprehensive objective functions will be introduced to consider nonlinear mechanical behavior, failure mechanisms, and damage evolution; the framework can be adapted for dynamic loading conditions, including fatigue and impact, to broaden its applicability in aerospace, automotive, and other high-performance engineering domains.

Acknowledgment

The authors acknowledge support from the Research Grants Council (RGC) of the Government of Hong Kong Special Administrative Region (HKSAR) with RGC/ECS Project (No. 26215824), HKUST research equipment development fund, and HKUST Bridge Gap Fund (BGF). This work was supported in part by the Project of Hetao Shenzhen-Hong Kong Science and Technology Innovation Cooperation Zone (HZQB-KCZYB-2020083). The help from the funding agency and the other researchers is sincerely appreciated.

CRediT authorship contribution statement

Molong Duan: Writing – review & editing, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation. **Ziyi Xu:** Visualization, Validation. **YANG Yuexin:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Data curation, Conceptualization.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used ChatGPT and Grammarly AI to perform grammar checks. After using these tools, the authors reviewed and edited the content as needed and took full responsibility for the content of the publication.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

References

- [1] P. Cheng, Y. Peng, S. Li, Y. Rao, A. Le Duigou, K. Wang, S. Ahzi, 3D printed continuous fiber reinforced composite lightweight structures: A review and outlook, *Compos B Eng.* 250 (2023) 110450, <https://doi.org/10.1016/j.compositesb.2022.110450>.
- [2] X. Wang, M. Jiang, Z. Zhou, J. Gou, D. Hui, 3D printing of polymer matrix composites: A review and prospective, *Compos B Eng.* 110 (2016), <https://doi.org/10.1016/j.compositesb.2016.11.034>.
- [3] Z. Gürdal, R.T. Haftka, P. Hajela, *Design and optimization of laminated composite materials*, John Wiley & Sons, 1999.
- [4] D. Zhao, W. Guo, Shape and performance controlled advanced design for additive manufacturing: a review of slicing and path planning, *J. Manuf. Sci. Eng.* 142 (2019), <https://doi.org/10.1115/1.4045055>.
- [5] D. Zhao, X. Zha, L. Zhou, Y. Xiong, Additive manufacturing of continuous fiber reinforced polymer composites: process planning and intelligent control, *J. Manuf. Process* 144 (2025) 170–189, <https://doi.org/10.1016/j.jmapro.2025.04.015>.
- [6] G. Fang, T. Zhang, Y. Huang, Z. Zhang, K. Masania, C.C.L. Wang, Exceptional mechanical performance by spatial printing with continuous fiber: curved slicing,

- toolpath generation and physical verification, *Addit. Manuf.* 82 (2024) 104048, <https://doi.org/10.1016/j.addma.2024.104048>.
- [7] Y. Li, C. Shi, X. Wang, S. Yuan, J. Zhu, W. Zhang, A mapping-based graded infill structure design method and continuous printing path planning, *Addit. Manuf.* 98 (2025) 104639, <https://doi.org/10.1016/j.addma.2025.104639>.
 - [8] G. Liu, W. Huang, Y. Wang, H. Ren, G. Zhang, L. Zhou, Y. Xiong, Stress field-aware infill toolpath generation for additive manufacturing of continuous fiber reinforced polymer composites, *Mater. Des.* 239 (2024) 112756, <https://doi.org/10.1016/j.matdes.2024.112756>.
 - [9] H. Zhang, J. Chen, D. Yang, Fibre misalignment and breakage in 3D printing of continuous carbon fibre reinforced thermoplastic composites, *Addit. Manuf.* 38 (2021) 101775, <https://doi.org/10.1016/j.addma.2020.101775>.
 - [10] X. Chi, J. Xue, L. Jia, J. Yao, H. Miao, L. Wu, T. Liu, X. Tian, D. Li, Machine learning-based online monitoring and closed-loop controlling for 3D printing of continuous fiber-reinforced composites, *Addit. Manuf. Front.* 4 (2025) 200196, <https://doi.org/10.1016/j.amf.2025.200196>.
 - [11] L. Lu, J. Hou, S. Yuan, X. Yao, Y. Li, J. Zhu, Deep learning-assisted real-time defect detection and closed-loop adjustment for additive manufacturing of continuous fiber-reinforced polymer composites, *Robot Comput. Integr. Manuf.* 79 (2023) 102431, <https://doi.org/10.1016/j.rcim.2022.102431>.
 - [12] Y. Yang, Y. Zhou, M. Duan, Contact-force-based closed-loop control of shell structure additive manufacturing with continuous-fiber-reinforced polymer composites, *J. Mater. Process Technol.* (2024) 118501, <https://doi.org/10.1016/j.jmatprotec.2024.118501>.
 - [13] J. Yao, A. Duongthipthewa, X. Xu, M. Liu, Y. Xiong, L. Zhou, Interlayer bonding improvement of PEEK and CF-PEEK composites with laser-assisted fused deposition modeling, *Compos. Commun.* 45 (2024) 101819, <https://doi.org/10.1016/j.coco.2024.101819>.
 - [14] J. Zhang, Z. Zhou, F. Zhang, Y. Tan, T. Yiwen, B. Yang, Performance of 3D-printed continuous-carbon-fiber-reinforced plastics with pressure, *Materials* 13 (2020) 471, <https://doi.org/10.3390/ma13020471>.
 - [15] Z. Duan, Y. Liu, J. Fan, K. Long, B. Xu, J. Zhu, J. Yan, Concurrent multi-material and multi-scale design optimization of fiber-reinforced composite material and structures for minimum structural compliance, *Compos Struct.* 311 (2023) 116796, <https://doi.org/10.1016/j.compstruct.2023.116796>.
 - [16] Y. Huang, X. Tian, Z. Zheng, D. Li, A.V. Malakhov, A.N. Polilov, Multiscale concurrent design and 3D printing of continuous fiber reinforced thermoplastic composites with optimized fiber trajectory and topological structure, *Compos Struct.* 285 (2022) 115241, <https://doi.org/10.1016/j.compstruct.2022.115241>.
 - [17] S. Li, W. Liu, Y. Mao, S. Hou, Numerical cross-scale optimization of homogenized composite laminates under impact loading, *Int J. Mech. Sci.* 245 (2023) 108108, <https://doi.org/10.1016/j.ijmecsci.2023.108108>.
 - [18] H. Wang, X.-Y. Zhong, H. Jia, L.-W. Zhang, H.-S. Liu, M.-C. Sun, T.-W. Liu, J.-B. Bai, S.-C. Ge, J.-W. Bao, Micromechanical modeling for longitudinal tensile property of unidirectional CFRP considering dispersion of fiber properties, *Compos Struct.* 339 (2024) 118081, <https://doi.org/10.1016/j.compstruct.2024.118081>.
 - [19] W. Zuo, X. Xie, R. Zhang, Y. Lu, T. Tang, L. An, J. Bai, Geometrically nonlinear topology and fiber orientation optimization of composite structures using membrane-embedded model, *ThinWalled Struct.* 203 (2024) 112250, <https://doi.org/10.1016/j.tws.2024.112250>.
 - [20] H. Guo, J. Wang, Z. Li, W. Zuo, J. Bai, Fiber path optimization of geometric nonlinear composite structures using a parametric level set method, *Eur. J. Mech. A/Solids* 114 (2025) 105769, <https://doi.org/10.1016/j.euromechsol.2025.105769>.
 - [21] H. Ren, D. Wang, G. Liu, D.W. Rosen, Y. Xiong, Concurrent optimization of structural topology and toolpath for additive manufacturing of continuous fiber-reinforced polymer composites, *Comput. Methods Appl. Mech. Eng.* 430 (2024) 117227, <https://doi.org/10.1016/j.cma.2024.117227>.
 - [22] S. He, P.-C. Ma, M. Duan, Continuous fiber path optimization in additive manufacturing: a gradient-based B-spline finite element approach, *Addit. Manuf.* 86 (2024) 104155, <https://doi.org/10.1016/j.addma.2024.104155>.
 - [23] T. Heitkamp, S. Kuschmitz, S. Girth, J.-D. Marx, G. Klawitter, N. Waldt, T. Vietor, Stress-adapted fiber orientation along the principal stress directions for continuous fiber-reinforced material extrusion, *Prog. Addit. Manuf.* 8 (2022) 541–559, <https://doi.org/10.1007/s40964-022-00347-x>.
 - [24] K. Sugiyama, R. Matsuzaki, A.V. Malakhov, A.N. Polilov, M. Ueda, A. Todoroki, Y. Hirano, 3D printing of optimized composites with variable fiber volume fraction and stiffness using continuous fiber, *Compos Sci. Technol.* 186 (2020) 107905, <https://doi.org/10.1016/j.compscitech.2019.107905>.
 - [25] T. Wang, N. Li, G. Link, J. Jelonnek, J. Fleischer, J. Dittus, D. Kupzik, Load-dependent path planning method for 3D printing of continuous fiber reinforced plastics, *Compos Part A Appl. Sci. Manuf.* 140 (2021) 106181, <https://doi.org/10.1016/j.compositesa.2020.106181>.
 - [26] X. Chen, G. Fang, W.-H. Liao, C.C.L. Wang, Field-Based Toolpath Generation for 3D Printing Continuous Fibre Reinforced Thermoplastic Composites, *Addit. Manuf.* 49 (2022) 102470, <https://doi.org/10.1016/j.addma.2021.102470>.
 - [27] T. Liu, S. Yuan, Y. Wang, Y. Xiong, J. Zhu, L. Lu, Y. Tang, Stress-driven infill mapping for 3D-printed continuous fiber composite with tunable infill density and morphology, *Addit. Manuf.* 62 (2023) 103374, <https://doi.org/10.1016/j.addma.2022.103374>.
 - [28] G. Zhang, Y. Wang, Z. Chen, X. Xu, K. Dong, Y. Xiong, Robot-assisted conformal additive manufacturing for continuous fibre-reinforced grid-stiffened shell structures, *Virtual Phys. Prototyp.* 18 (2023), <https://doi.org/10.1080/17452759.2023.2203695>.
 - [29] Z. Hou, X. Tian, J. Zhang, Z. Zheng, L. Zhe, D. Li, A.V. Malakhov, A.N. Polilov, Optimization design and 3D printing of curvilinear fiber reinforced variable stiffness composites, *Compos Sci. Technol.* 201 (2021) 108502, <https://doi.org/10.1016/j.compscitech.2020.108502>.
 - [30] F. Fernandez, W.S. Compel, J.P. Lewicki, D.A. Tortorelli, Optimal design of fiber reinforced composite structures and their direct ink write fabrication, *Comput. Methods Appl. Mech. Eng.* 353 (2019) 277–307, <https://doi.org/10.1016/j.cma.2019.05.010>.
 - [31] C. Luo, F. Ferrari, J.K. Guest, Design optimization of advanced tow-steered composites with manufacturing constraints, *Compos B Eng.* 284 (2024) 111739, <https://doi.org/10.1016/j.compositesb.2024.111739>.
 - [32] S. Khan, K. Fayazbakhsh, Z. Fawaz, M. Arian Nik, Curvilinear variable stiffness 3D printing technology for improved open-hole tensile strength, *Addit. Manuf.* 24 (2018) 378–385, <https://doi.org/10.1016/j.addma.2018.10.013>.
 - [33] N. Kumekawa, Y. Mori, H. Tanaka, R. Matsuzaki, Experimental evaluation of variable thickness 3D printing of continuous carbon fiber-reinforced composites, *Compos Struct.* 288 (2022) 115391, <https://doi.org/10.1016/j.compstruct.2022.115391>.
 - [34] Y. Huang, X. Tian, L. Wu, A.A. Zia, T. Liu, D. Li, Progressive concurrent topological optimization with variable fiber orientation and content for 3D printed continuous fiber reinforced polymer composites, *Compos B Eng.* 255 (2023) 110602, <https://doi.org/10.1016/j.compositesb.2023.110602>.
 - [35] Z. Yang, K. Fu, Z. Zhang, J. Zhang, Y. Li, Topology optimization of 3D-printed continuous fiber-reinforced composites considering manufacturability, *Compos Sci. Technol.* 230 (2022) 109727, <https://doi.org/10.1016/j.compscitech.2022.109727>.
 - [36] Y. Huang, X. Tian, Z. Zheng, D. Li, A.V. Malakhov, A.N. Polilov, Multiscale concurrent design and 3D printing of continuous fiber reinforced thermoplastic composites with optimized fiber trajectory and topological structure, *Compos Struct.* 285 (2022) 115241, <https://doi.org/10.1016/j.compstruct.2022.115241>.
 - [37] E.M. Stein, R. Shakarchi, *Complex analysis*, Princeton University Press, 2010.
 - [38] Y. Zhou, L. Gao, H. Li, Graded infill design within free-form surfaces by conformal mapping, *Int J. Mech. Sci.* 224 (2022) 107307, <https://doi.org/10.1016/j.ijmecsci.2022.107307>.
 - [39] X. Jiang, W. Huo, C. Liu, Z. Du, X. Zhang, X. Li, X. Guo, Explicit layout optimization of complex rib-reinforced thin-walled structures via computational conformal mapping (CCM), *Comput. Methods Appl. Mech. Eng.* 404 (2023) 115745, <https://doi.org/10.1016/j.cma.2022.115745>.
 - [40] T.W. Meng, G.P.T. Choi, L.M. Lui, TEMPO: Feature-endowed teichmüller extremal mappings of point clouds, *SIAM J. Imaging Sci.* 9 (2016) 1922–1962, <https://doi.org/10.1137/15M1049117>.
 - [41] K. Xu, X. Hao, J. Lin, Automated fibre placement path generation for complex surfaces via digital image deconvolution algorithm, *Compos Part A Appl. Sci. Manuf.* 163 (2022) 107246, <https://doi.org/10.1016/j.compositesa.2022.107246>.
 - [42] Y. Yang, Y. Zhou, M. Duan, Contact-Force-Based Closed-Loop Control of Multi-Axis Additive Manufacturing with Continuous-Fiber-Reinforced Polymer, in: *Manufacturing Science and Engineering Conference 2025*, Greenville, USA, 2025.
 - [43] M.C. Serna Moreno, S. Horta Muñoz, Mechanical response of $\pm 45^\circ$ angle-ply CFRP plates under low-velocity impact and quasi-static indentation: Influence of the multidirectional strain state, *Compos Sci. Technol.* 194 (2020) 108145, <https://doi.org/10.1016/j.compscitech.2020.108145>.
 - [44] Ç. Kahya, O. Tunçel, O. Çavuşoğlu, K. Tüfekci, Thermal annealing optimization for improved mechanical performance of PLA parts produced via 3D printing, *Polym. Test.* 144 (2025) 108735, <https://doi.org/10.1016/j.polymertesting.2025.108735>.
 - [45] I. Ouchte, J. Chafiq, M. Ait El Fqih, H. Chakir, Effect of thermal treatment on mechanical properties and thermogravimetric analysis of laminate composite jute/epoxy, *Mater. Today Proc.* 66 (2022) 135–139, <https://doi.org/10.1016/j.matpr.2022.04.209>.
 - [46] T. Zhang, T. Liu, N. Dutta, Y. Chen, R. Su, Z. Zhang, W. Wang, C.C.L. Wang, Toolpath generation for high density spatial fiber printing guided by principal stresses, *Compos B Eng.* 295 (2025) 112154, <https://doi.org/10.1016/j.compositesb.2025.112154>.
 - [47] J. Xiang, P. Cheng, K. Wang, Y. Wu, Y. Rao, Y. Peng, Interlaminar and translaminar fracture toughness of 3D-printed continuous fiber reinforced composites: A review and prospect, *Polym. Compos* 45 (2024) 3883–3900, <https://doi.org/10.1002/pc.28065>.
 - [48] G.D. Goh, V. Dikshit, A.P. Nagalingam, G.L. Goh, S. Agarwala, S.L. Sing, J. Wei, W. Y. Yeong, Characterization of mechanical properties and fracture mode of additively manufactured carbon fiber and glass fiber reinforced thermoplastics, *Mater. Des.* 137 (2018) 79–89, <https://doi.org/10.1016/j.matdes.2017.10.021>.