



Property-controllable process planning for multi-axis hybrid manufacturing of complex metal parts

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ABSTRACT

While hybrid manufacturing (HM), which integrates additive manufacturing (AM) and subtractive manufacturing (SM) processes, facilitates the creation of complex cavities with high surface quality, optimizing the sequence of AM and SM operations in multi-axis HM systems poses significant challenges, especially for directed energy deposition (DED). Existing methods lack efficient handling of the multi-axis particularity of DED on physical and geometric factors, and the flexibility is restricted to sequence planning at the feature-level or layer-level. This study focuses on multi-axis DED-based HM and optimizes the path-level additive-subtractive (A-S) alternation sequence under the collision-free and physical properties-related constraints by formulating the optimization as a sequential ordering problem. Firstly, the input model is sliced into a set of layers by which the AM and SM dependency digraphs are constructed to capture the precedence constraints. Secondly, the multi-axis printability of each AM layer is evaluated to incorporate the collision-free constraint, enabling the optimization of a pure AM sequence. Next, at each cutter location point on each cutting path, the concept of tool axis accessibility heatmap is proposed to facilitate the optimization of the tool axis for enhanced machinability and motion smoothness. Finally, printability and machinability are utilized to build a comprehensive A-S dependency digraph to optimize the A-S alternation, whose directed edges reflect the collision constraint, while the edge weights can be adjusted for property-controllable sequence planning. Simulations and experiments on exemplary parts demonstrate that the proposed HM approach achieves a material waste reduction of more than 50% compared to conventional pure machining. Furthermore, the proposed path-level HM sequence planning method reduces machining segments by up to 30% over layer-level approaches.

1. Introduction

Complex components, such as exhaust manifolds, combustion chambers, and blades with conformal cooling channels, are essential in automotive, energy, and aerospace industries, yet pose significant manufacturing challenges due to their deep and narrow cavities. Conventional methods like casting struggle with part complexity, cost, and inefficiency, while partitioning and welding often compromise precision and process quality (Freeman, 2012). Machining, though precise, incurs high material and time costs, especially for expensive alloys, and faces cutter collision risks in complex internal cavities (Chen et al., 2018;

Feldhausen et al., 2022). On the other hand, additive manufacturing (AM) offers flexibility and material efficiency, but is limited by insufficient accuracy and poor surface finish (Pragana et al., 2021).

To leverage the strengths of both additive and subtractive (i.e., machining) methods, hybrid manufacturing (HM) strives to integrate AM and conventional subtractive machining (SM) into one single embodiment, wherein the AM and SM are alternately performed. Through a rigorously optimized HM process, a complex component that originally was very difficult or even impossible to machine due to the tool accessibility constraint can now be manufactured with high surface quality and low time/material consumption. Regarding the metallic AM

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constituents of HM technology, laser-powder directed energy deposition (LP-DED) (Thompson et al., 2015) and powder bed fusion (PBF) (Wang et al., 2023) predominate; specifically, LP-DED facilitates precise deposition control and multi-axis capabilities, which is useful for repairing of an existing part (Kanishka and Acherjee, 2023; Le et al., 2017; Ma et al., 2025). This paper is thus focused on LP-DED, which is hereinafter referred to as DED for short, wherever no confusion arises.

The key scientific issue of process planning of HM is to plan and optimize the alternating sequence between AM and SM under the constraints of precedence preservation, collision avoidance, and physical property requirements (Chen et al., 2020). A typical HM sequence can be represented as $\{A_1, S_1, A_2, S_2, \dots, A_n, S_n\}$, where “A” and “S” stand for the AM and SM operations (sub-processes) respectively, and n is the number of additive-subtractive (A-S) alternations between the two. Intuitively, to reduce the collision risk of HM as much as possible, one can increase the alternation number n – an extreme case is machining each layer right after it is printed. However, frequent alternation between AM and SM introduces at least four drawbacks: (1) the thermal distortion of the workpiece compromises dimensional accuracy and induces local shrinkage at the A-S interfaces (Nguyen et al., 2020); (2) the re-melting at the A-S interfaces requires additional finishing effort (Nishiyama et al., 2023); (3) the high-temperature oxidative discoloration and contamination from powder spatter; and (4) heat treatment is typically required after the AM process to reduce the residual stress and cutting force (Lalegani Dezaki et al., 2022), which prolonging the manufacturing time. Consequently, the primary objective of the sequence optimization algorithm is to minimize the number of alternations to maximize manufacturing efficiency and surface quality while adhering to collision-avoidance and physical constraints related to the AM nozzle and SM cutter. Beyond sequence planning, HM constraints can be imposed on topology optimization (Liu and To, 2016; Xu et al., 2024) which is another potential topic.

1.1. Related work

The integration of AM and SM has been investigated from different aspects. Ren et al. (2010) decomposed the part based on different additive orientations and then built up the entire part prior to a final machining stage. Kerbrat et al. (2010) employed the manufacturability index calculated during the part design stage to decompose the part to improve the manufacturability of the existing model. Joshi and Anand (2017) calculated potential manufacturing complexity during the product design phase and decomposed the model into additive and subtractive parts by an optimized segmentation plane for minimal complexity. However, the above methods are limited to a single post-processing subtractive operation following the completion of the AM phase and they fail to address the A-S sequence optimization subject to the collision-avoidance constraint, which presents severe limitations for complex geometries, particularly those featuring deep and curved cavities.

Regarding A-S alternation, Behandish et al. (2018) utilized manufacturing primitives to formulate a Boolean algebra, decomposing a part into standardized intersection terms for manufacturability analysis and three-axis machining, albeit without optimizing the HM sequence. Zheng et al. (2020) employed the level-set method for the automatic extraction of 2.5-axis additive-subtractive features, optimizing the feature-level alternating sequence by minimizing a cost function under priority constraints. This feature-level mode decomposition is also employed in remanufacturing. Zhu et al. (2013) proposed an HM process called iAtractive by defining a series of manufacturability indicators to sequence features based on their hierarchical relationships. Le et al. (2018) presented a systematic HM strategy for direct part reuse leveraging manufacturing features, process knowledge, technological requirements, and available resources. Chen and Frank (2019) optimized the part setups for both SM and AM to minimize the support structure of AM and the machining time, starting from machining the

base plate, followed by the deposition on the machined surface as well as finish machining without further A-S alternation. Rossi and Lanzetta (2020) extracted AM and SM features from computer aided design (CAD) models, scheduling them using metaheuristic algorithms to address operational flexibility, feature precedence, and sequencing complexities. Kerbrat et al. (2011) developed a design-for-manufacturing system to evaluate quantitative HM indices on each module, which were used to determine the machining or additive processes. Kwon and Oh (2023) proposed a recursive volume decomposition method with decision criteria to optimize the material cost and time of the HM process. In general, many existing methodologies are focused on the feature level by (manual) feature extraction without AM/SM paths, or they typically perform (manual) feature extraction and AM/SM path planning separately. This feature-level planning tends to yield sub-optimal decomposition results since the collision constraint regarding the nozzle and cutter were not considered in the CAD feature extraction phase. Furthermore, the AM/SM paths themselves may affect the initial feature decomposition results in turn, leading to a potential 'greedy lock' issue when decomposition is terminated prematurely. Consequently, a paradigm shift from feature-level to layer-level planning is required, wherein AM layers serve as the basic units for alternation sequence planning in HM.

To determine the HM sequence at the layer level, Ruan et al. (2005) divided the printing process into two phases: printing uniform layers with constant thickness and then removing excess material to form non-uniform layers, which results in frequent AM-SM alternation after each layer. To reduce the number of A-S alternations, Xiong et al. (2009) proposed an additive-subtractive alternating method for simple parts, first depositing the part with a fixed thickness, and then performing milling when the thickness exceeds a threshold. Abe et al. (2023) determined the three-axis AM height for each layer by simulating a non-colliding cutting process, thus establishing the alternating sequence, but this method only considers the interference issue from the height perspective under the three-axis scheme. Urbanic et al. (2022) decomposed a part manually and planned discrete multidirectional AM paths followed by machining in a simulation environment. Considering that AM is a volumetric operation while SM is a surface operation, Zhong et al. (2023b) formulated the HM sequence planning problem as a volume-and-surface-co-decomposition problem which is solved by a top-down block decomposition algorithm.

The majority of studies on layer-level path planning are focused on only three-axis printing and/or machining on the input geometry. For more complicated parts, the input model typically requires decomposition into discrete sub-components, each of which is printed in one direction. The emerging multi-axis technologies enhance the versatility of HM by incorporating additional rotational degrees of freedom (DOFs) into the kinematics of either the tool head or the worktable. In the context of SM, multi-axis kinematics mitigate collision risks since more accessible tool orientations can be realized (Chen et al., 2020; Wang et al., 2025). For AM, both the printing direction and layer thickness of a slice become variable parameters during printing, which can effectively reduce the need for support structures (Bhatt et al., 2020), enhance surface quality, and improve mechanical properties (Chen and Frank, 2019). Owing to these advantages, Li et al. (2018) and Keating and Oxman (2013) built a 6-DOF robotic platform equipped with changeable heads (AM and SM) for efficient multi-axis HM. Chen et al. (2020) proposed a top-down algorithm to optimize layer heights governed by nozzle printability and cutter machinability constraints within a multi-axis scheme, thus establishing the optimal HM alternation sequence to improve overall efficiency.

All the aforementioned works are confined to polymer-based (i.e., non-metal) AM, whereas metallic processes, specifically LP-DED, impose additional complexities on multi-axis kinematic schemes and A-S alternation (He et al., 2024a). Specifically, unlike the polymer-based printing shown in Fig. 1(a), the blown powder-based DED is prone to the leakage of the laser beam if its orientation is simply aligned with the layer

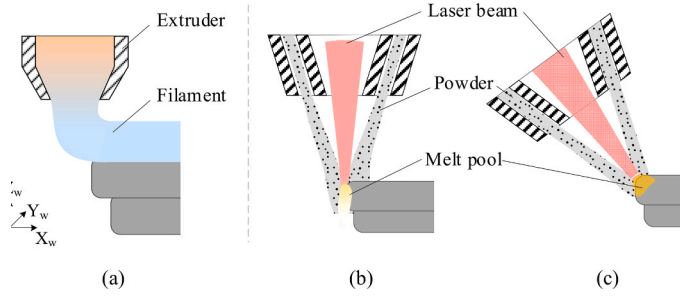


Fig. 1. The necessity of continuously varying the nozzle axis in DED (He et al., 2024a): (a) in polymer-based printing, the printing layer adheres to the previous one which allows for a larger overhang; (b) the fixed nozzle axis causes the leakage of powder and laser beam; (c) varied nozzle axis stabilizes the deposition due to the correct formation of the melt pool on the previous layer.

normal (Fig. 1(b)), which jeopardizes the correct forming of the melt pool. Therefore, for LP-DED, the nozzle must be inclined towards the part profile such that the melt pool on the previous layer can be formed successfully (Fig. 1(c)). In other words, the nozzle orientation must be continuously varied in one layer, conforming to the model's geometry (Campocasso et al., 2023; Plakhotnik et al., 2019). From the cutting aspect, deep cavity features for HM also necessitate the continuous variation of the cutter axis to avoid any potential collision (He et al., 2024b). These requirements on the tool axes of AM and SM bring about challenges to the sequence planning of multi-axis HM.

Beyond geometric accessibility constraints, the thermal effect and shape control of metallic AM pose challenges to A-S alternation as well. Nishiyama et al. (2023) optimized the alternation sequence by introducing the effects of remelting penetration such that AM and SM blocks need not have the same length, although only the SM toolpath simulation without collision was considered in terms of manufacturability constraints. A recent layer-level HM sequence planning method (Chen et al., 2025) in the authors' group specifies the distance of the remelting as a fixed length while not thoroughly investigating the mechanism of priority constraint and remelting mitigation. In view of the physical characteristics of PBF, Wang et al. (2025, 2023) compensated the AM path for PBF to retain the finishing allowance, extracted HM features from the CAD model under the constraints of the tool accessibility and PBF characteristics, and then planned multi-layer cutting paths by a boundary-guided method.

1.2. Research motivations

Notwithstanding the above process planning methods for multi-axis HM, the following issues remain:

- (1) The advantages of multi-axis AM and SM for the sequence planning of HM have yet to be fully exploited. For AM, the deposition mechanism of LP-DED requires multi-axis motion to ensure stable deposition; for SM, multi-axis motion is needed to expand the tool-accessible region. The geometric effect of multi-axis capabilities on A-S alternation sequences requires a further systematic investigation.
- (2) The particularity of LP-DED introduces new (or pre-determined) physical mechanisms that impose constraints or preferences on an A-S sequence. The interactions between these physical factors and geometric ones need to be studied, which also present significant integration challenges for existing top-down search algorithms primarily due to the high sensitivity of seed sub-component selection. Since the HM sequence optimization problem has an extremely large solution space, a general optimization framework of HM incorporating multiple physical constraints/preferences is worth studying.

- (3) The basic unit for HM should advance beyond the layer level to the path level, thus enabling greater flexibility. The machining paths belonging to 1 AM layer may have multiple connected components that are not necessarily machined in one alternation. As shown in Fig. 2(a), the three tube-like blocks are sequentially deposited, but their outer surfaces should be machined contiguously to minimize the staircase effect at the A-S interfaces (Fig. 2(b)).

In this paper, to address the above issues, a general property-controllable sequence planning framework for DED-based multi-axis HM is proposed, which can: (1) adhere to collision constraints through multi-axis printability and machinability analysis tailored for DED; (2) enable property-controllable sequence planning satisfying the physical or design requirements; and (3) perform path-level sequence optimization, offering greater flexibility and adaptability. The entire HM sequence planning is formulated as a sequential ordering problem (SOP). A novel finding from this study is that solving a sub-problem for pure AM sequence planning significantly streamlines the broader HM sequence planning problem. The overall pipeline for HM sequence planning is summarized as follows.

Firstly, the given input model is partitioned and sliced into a set of layers, based on which an AM digraph is built, and the connected contours of each layer are extracted to form an SM digraph (Section 2.1), capturing the AM and A-S dependency (or precedence) constraints (Section 2.2). Then, the nozzle's multi-axis printability on each AM layer is computed, which is further incorporated into the AM directed graph (digraph) for optimizing a pure AM sequence (Section 3.1). Next, on each cutter location (CL) point of each basic cutting path, many tool axes on a Gaussian sphere are sampled and ranked via the previously obtained AM sequence, followed by tool axis optimization on each basic cutting path (Section 3.2). With the cutter axes determined, the machinability of each cutting path is thus evaluated, and the collision-free precedence relation between AM and SM operations is obtained, which is augmented to the total A-S digraph for HM sequence optimization (Section 2.3 & 2.4). Furthermore, the property-controllable capabilities of the HM process are realized by modulating digraph edge weights or incorporating additional edges (Section 3.3). Finally, the proposed methodology is validated through both computer simulations and physical experiments, as reported in Section 4.

2. Problem formulation for graph-based HM

The watertight input geometry, represented as a triangular mesh $\Omega = (V, F)$, is initially partitioned and sliced into a set of layers, based on which the basic units for AM and SM are respectively identified to form the underlying graph structure. The constraints and preferences of HM are encoded into a generalized weighted digraph for the optimization of the HM sequence.

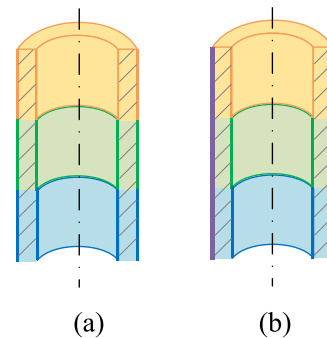


Fig. 2. The basic unit for HM needs to be evolved from (a) the layer level to (b) the path level for more flexibility.

2.1. Graph-based manufacturing basic unit for HM

In the first step of the proposed pipeline, the surface mesh Ω with a complex structure is partitioned into several subparts, e.g., $\Omega = \Omega_1 \cup \Omega_2$ as shown in Fig. 3(a), which is typically accomplished by relying on the skeleton curves of Ω , some guide curves assigned manually, or other feature extraction methods. The planar cross profiles resulting from each partition plane are then filled by the hole-filling algorithm to maintain the water tightness of each sub-mesh. For each simple subpart, say Ω_1 , an adaptive slicing method using non-parallel planes is employed (Eisenbarth et al., 2021) whose layer thickness range is $[\delta t_{\min}, \delta t_{\max}]$, yielding a sequence of slices $\{S_i\}$ and their corresponding layers $\{L_i\}$, as illustrated in Fig. 3(a) (the layer spacing is exaggerated for better visibility). Each layer L_i is composed of four kinds of portions: the bottom slice S_i , the upper slice S_{i+1} , the lateral surface $S_i^{i+1} \in \Omega$ sandwiched between S_i and S_{i+1} , and the auxiliary lateral surface $S_p \notin \Omega$ inherited from the partition planes, such that the total layer $L_i = S_i \cup S_i^{i+1} \cup S_{i+1} \cup S_p$, as shown in Fig. 4(a).

Each layer L_i naturally acts as a basic manufacturing primitive for AM since after partition, it is a topologically simply connected volume, as exemplified by L_4 , L_8 and L_{11} in Fig. 3(a). The volume of each layer can be printed by generating internal toolpaths that conform to the layer's boundary. As the layer thickness is normally very small, the volume-filling task of the layer is simplified to filling the bottom slice S_i of L_i with programmed varied thicknesses (He et al., 2024a). The AM units in Fig. 3(a) are further interconnected to construct a digraph in Fig. 3(b) based on the layer adjacency relationship, in which each node N_i^a denotes an AM layer and the arrow means that the target node must be printed after the source node. At any junction between subparts, the adjacent relation still holds, e.g., L_6 to L_9 in Fig. 3(a) touch the bottom of L_{12} , meaning that L_{12} can only be printed after L_6 to L_9 are printed.

The basic manufacturing primitive for SM, conventionally, is still the deposition layers $\{L_i\}$ in which the lateral surfaces S_i^{i+1} inherited from the original mesh Ω requires machining. It is noted that S_i , S_{i+1} and S_p do not need to be machined since they will be covered by the deposited materials later. For complicated geometries, especially with cavities, however, the lateral surfaces of a layer L_i may comprise multiple disjoint connected components. For example, in the enlarged view of Fig. 3(a), both L_4 and L_8 feature two distinct connected surfaces (in blue and green colors), denoted as “4, 1” and “4, 2”, “8, 1” and “8, 2”, respectively. In the conventional setting, the two lateral surfaces of each layer are

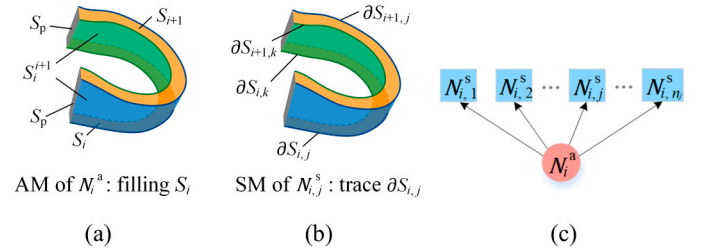


Fig. 4. The basic HM unit for AM and SM, respectively: (a) the AM node N_i^a of layer i ; (b) the SM node $N_{i,j}^s$ for which the cutter traces the contour boundary $\partial S_{i,j}$; (c) each AM node is split into several SM nodes $\{N_{i,j}^s\}_{j=1,2,\dots,n_j}$.

machined simultaneously, which constrains the sequence of cutting. In reality, the lateral surfaces of a layer do not have to be machined at one time. Specifically, for the tube structure in Fig. 3, the outside surface of Ω can be machined as a group after the whole part has been built, which means the SM sequence is independent of the AM unit. Therefore, in this paper, a simply connected lateral surface is regarded as a basic unit for SM, and in practice, considering that layers are very thin, the basic unit for SM is a *simply connected (planar) curve* on the surface of Ω for the potential cutting tool path, as shown by the curves in different colors in Fig. 3(a). Fig. 4(b) illustrates the construction of a simply connected curve, in which the contour boundary ∂S_i of slice S_i has two simply connected curves: $\partial S_{i,j}$ and $\partial S_{i,k}$. Taking $\partial S_{i,j}$ for an example, it is a set of points that are located on the j^{th} connected component of ∂S_i and the original mesh, i.e.,

$$\partial S_{i,j} = \{p | p \in \{\partial S_i\}_j \& p \in \Omega\}, \quad \partial S_i = \cup_j \partial S_{i,j} \quad (1)$$

The subtractive node for the basic machining unit is denoted as $N_{i,j}^s$, corresponding to the simply connected boundary curve $\partial S_{i,j}$ (the i^{th} layer's j^{th} curve), whose connecting relation is shown in Fig. 3(c), which can also be deduced from the AM digraph in Fig. 3(b). Conceptually, each AM node N_i^a is split into several SM nodes $\{N_{i,j}^s\}_{j=1,2,\dots,n_j}$ whose total number n_j is determined by the number of simply connected curves of ∂S_i (Fig. 4(c)). Unlike the adjacency criteria between AM nodes, two adjacent SM nodes $N_{i,j}^s$ and $N_{k,l}^s$ should satisfy the following two criteria:

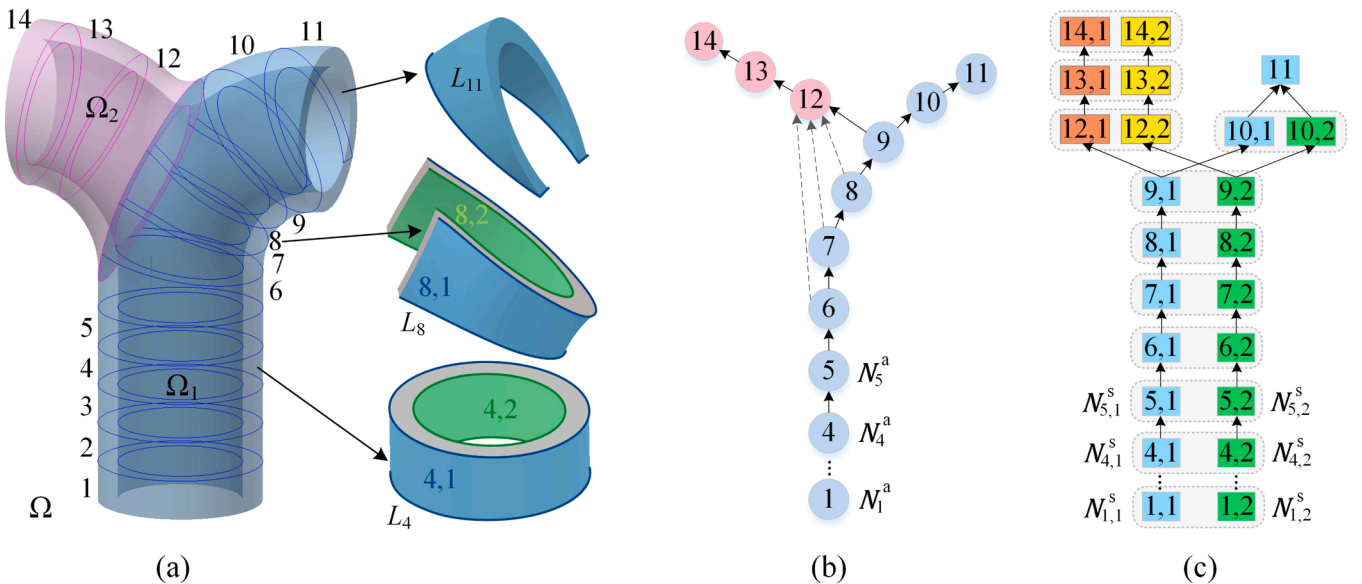


Fig. 3. The basic unit of HM: (a) adaptive slicing of the input model Ω into many layers; (b) AM digraph; (c) SM digraph.

- (1) they belong to adjacent AM nodes (i.e., N_i^a and N_k^a are adjacent), and
- (2) the associated curve ∂S_{ij} has at least one point $\mathbf{p} \in \partial S_{ij}$ from which a geodesic curve on Ω is traced along the tangent direction $\mathbf{d}$ perpendicular to ∂S_{ij} at $\mathbf{p}$, the curve first intersects ∂S_{kl} at $\mathbf{p}' \in \partial S_{kl}$, and the geodesic curve $\widehat{\mathbf{pp}'} \in \Omega$ & $\widehat{\mathbf{pp}'} < 2\delta t_{\max}$ (Li et al., 2021), as shown in Fig. 5.

According to the adjacency relation of AM nodes in Fig. 3(b) and the criteria of adjacency of SM nodes, an SM digraph is constructed, as shown in Fig. 3(c). For example, the SM node $N_{4,1}^s$ points towards $N_{5,1}^s$ while node $N_{4,2}^s$ points towards $N_{5,2}^s$; $N_{4,1}^s$ remains disconnected to $N_{5,2}^s$ as the geodesic distance in-between exceeds the preset machining interval $2\delta t_{\max}$. Apart from the adjacency relation within AM nodes or SM nodes, the connections between AM and SM nodes are far from trivial, which will be discussed in the next subsection.

2.2. Hybrid manufacturing constraints

The previous subsection established fundamental internal connections for AM and SM nodes, respectively, generating two distinct digraphs. In this subsection, the inter-digraph connections between the AM and SM digraphs are considered by introducing dependency and collision-free constraints, respectively, forming a comprehensive precedence graph for subsequent graph-based HM sequence optimization.

2.2.1. Dependency preservation

The dependency constraint can be classified into two categories: AM depending on AM, and SM depending on AM. In specifics, as shown in Fig. 6(a), the AM layer N_{i+1}^a (a.k.a. AM node) can only be printed after its topological predecessors N_i^a , N_{i-1}^a , ... have been printed already. This AM dependency can be naturally inherited from the AM digraph in Fig. 3(b). As shown in Fig. 6(b), a layer can only be machined after it has already been printed, which means the AM node N_i^a must point to its corresponding SM nodes $N_{i,*}^s$ (* represents an arbitrary sub-index of i). This dependency of SM on AM can be naturally inherited from the node split in Fig. 4(c). The integrated dependency preservation constraint is summarized in Fig. 6(c), in which the AM nodes (circular) and SM nodes (square) are on the left and right, respectively. The AM nodes must point to all their successors *recursively*, and also must point to ALL their corresponding SM nodes. It is noted that other dependencies of SM on AM can be inferred as well, e.g., $N_i^a \rightarrow N_{i+1,*}^s$ since $N_i^a \rightarrow N_{i+1}^a \rightarrow N_{i+1,*}^s$.

2.2.2. Collision avoidance

Apart from the dependency constraint, collision avoidance must be considered, which can also be classified into four categories: AM should not collide with AM, AM should not collide with SM, SM should not collide with AM, and SM should not collide with SM, which are detailed below. The specific collision check computational algorithm will be detailed in Section 3, and in this section, it is assumed that the collision status is predetermined.

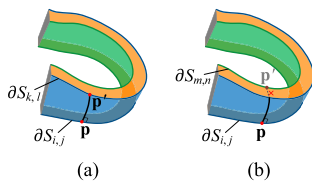


Fig. 5. The adjacency relation between two SM nodes: (a) $N_{i,j}^s$ is adjacent to $N_{k,l}^s$ since one point $\mathbf{p} \in \partial S_{ij}$ can be traced to ∂S_{kl} without leaving Ω and vice versa; (b) $N_{i,j}^s$ is NOT adjacent to $N_{m,n}^s$ since no point $\mathbf{p} \in \partial S_{ij}$ can be traced to ∂S_{mn} without leaving Ω and vice versa.

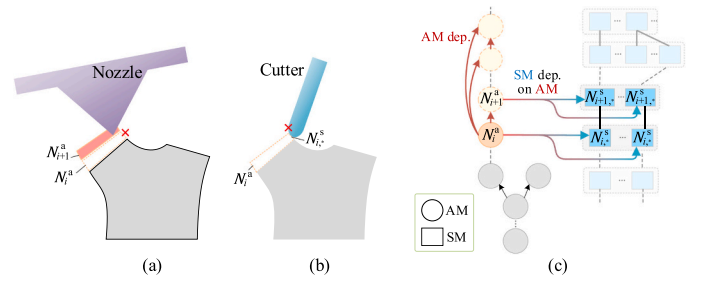


Fig. 6. The dependency constraints should be preserved: (a) a layer can only be printed after all its predecessors have been printed already (i.e., AM dependency); (b) a layer can only be machined after it has been printed already (i.e., SM depends on AM); (c) the precedence graph consisting of the dependency of (a) and (b).

- (1) *AM should not collide with AM.* As depicted in Fig. 7(a), assuming that the printing paths and nozzle orientations of the AM node N_i^a (i.e., layer N_i^a) are given, when printing each point $\forall \mathbf{p} \in N_i^a$, the nozzle with axis $\mathbf{t}$ should not intersect with another AM node N_j^a :

$$\mathcal{T}_a(\mathbf{p}, \mathbf{t}) \cap N_j^a = \emptyset, \quad \forall \mathbf{p} \in N_i^a \quad (2)$$

where $\mathcal{T}_a(\mathbf{p}, \mathbf{t})$ denotes the AM nozzle assembly located at $\mathbf{p}$ with axis $\mathbf{t}$. If this AM-AM collision-free condition in Eq. (2) is not satisfied, node N_i^a must be printed before N_j^a to ensure feasibility (Fig. 7(b)). That is, a new directed edge pointing from N_i^a to N_j^a , i.e., $N_i^a \rightarrow N_j^a$ is added to the digraph.

- (2) *AM should not collide with SM.* Similar to the configuration in (1), when printing the AM node N_i^a , the nozzle located at $\mathbf{p}$ with axis $\mathbf{t}$ must not intersect with the SM nodes $N_{j,*}^s$ (the SM nodes of the AM node N_j^a). This collision check requires no explicit computation but a simple logical inference from the dependency of AM on SM (discussed in the previous subsection). Since layer j can only be machined after it is printed (i.e., $N_j^a \rightarrow N_{j,*}^s$), it is guaranteed that $N_i^a \rightarrow N_{j,*}^s$ as long as $N_i^a \rightarrow N_j^a$.
- (3) *SM should not collide with AM.* As shown in Fig. 7(c), assuming that the machining paths and cutter orientations of the SM node $N_{i,j}^s$ of layer N_i^a are given, when machining each point $\forall \mathbf{p} \in N_{i,j}^s$, the cutter with axis $\mathbf{t}$ should not intersect with another AM node N_k^a :

$$\mathcal{T}_s(\mathbf{p}, \mathbf{t}) \cap N_k^a = \emptyset, \quad \forall \mathbf{p} \in N_{i,j}^s \quad (3)$$

where $\mathcal{T}_s(\mathbf{p}, \mathbf{t})$ denotes the cutter assembly located at $\mathbf{p}$ with axis $\mathbf{t}$. If the collision-free condition in Eq. (3) is violated, node $N_{i,j}^s$ must be machined before N_k^a . That is, a new directed edge pointing from $N_{i,j}^s$ to N_k^a , i.e., $N_{i,j}^s \rightarrow N_k^a$, is added to the digraph to enforce this precedence, as shown in Fig. 7(d).

- (4) *SM should not collide with SM.* Similar to the configuration in (3), when machining the SM node $N_{i,j}^s$, the cutter located at $\mathbf{p}$ with axis $\mathbf{t}$ should not intersect with the SM nodes $N_{k,*}^s$ (the SM nodes of the AM node N_k^a). This collision check needs no computation as well, but a simple inference based on the dependency of AM on SM (discussed in the previous subsection). Since layer k can only be machined after it is printed ($N_k^a \rightarrow N_{k,*}^s$), it is guaranteed that $N_{i,j}^s \rightarrow N_{k,*}^s$ as long as $N_{i,j}^s \rightarrow N_k^a$.

2.3. Hybrid manufacturing preferences

The hybrid manufacturing constraints in Section 2.2 give mandatory precedence to HM nodes by adding directed edges in AM and SM digraphs, respectively, as shown in Fig. 3(c)&(d). Apart from these compulsory constraints in terms of dependency and collision, other soft

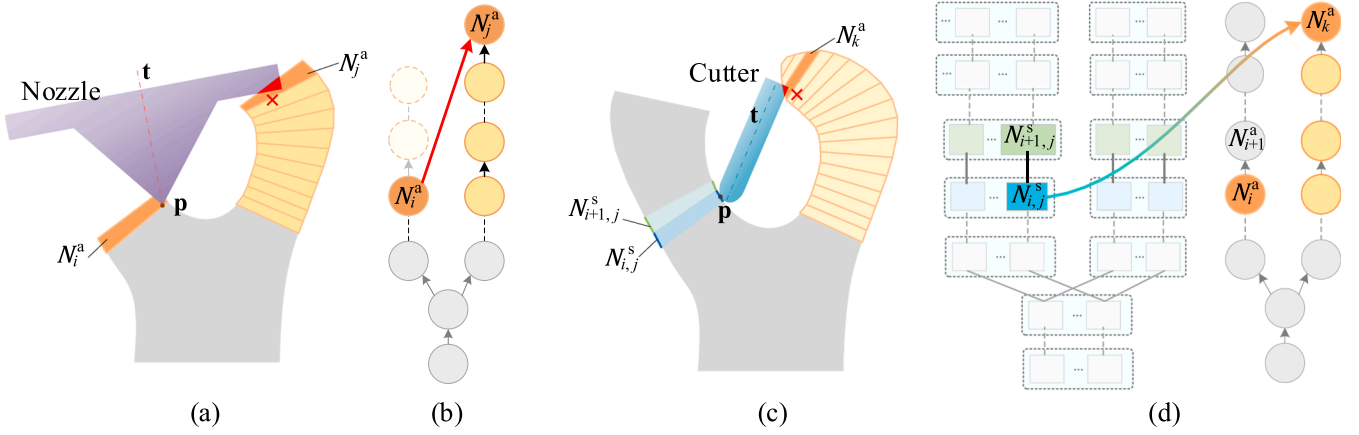


Fig. 7. Collision avoidance constraint between nodes: (a) when printing N_i^a , the nozzle should not collide with N_j^a ; (b) a new directed edge $N_i^a \rightarrow N_j^a$ is added since N_i^a must be printed before N_j^a ; (c) when machining $N_{i+1,j}^s$, the cutter should not collide with N_k^a ; (d) a new directed edge $N_{i+1,j}^s \rightarrow N_k^a$ is added since $N_{i+1,j}^s$ must be machined before N_k^a .

preference metrics should be considered, such as minimizing (1) the number of AM-SM alternations; (2) the number of non-productive air movements in AM; and (3) the number of non-productive air movements in SM. These metrics serve as the preference function in HM sequence planning, as detailed below.

Minimizing the number of AM-SM alternations is designated as the primary objective, which is accomplished by assigning a very large number d_{as} (say 1×10^5) to the cost of changing the current tool from nozzle to cutter. As shown in Fig. 8(c), after printing N_k^a , the nozzle is stored in the magazine and then a cutter is used to machine the SM nodes $N_{i+1,j}^s$ of N_i^a , which amounts to assigning d_{as} to the cost of the *undirected* edge between N_k^a and $N_{i+1,j}^s$ (Fig. 8(d)) if the precedence and collision-free constraints in Section 2.2 are satisfied. This can also be represented in a cost matrix as $C(N_k^a, N_{i+1,j}^s) = C(N_{i+1,j}^s, N_k^a) = d_{as}$, in which $C(x, y)$ denotes the cost from node x to node y .

In order to maintain high AM and SM efficiency when determining the sequence of nodes, the algorithm encourages the contiguous

traversal of adjacent nodes within the same process, in which the adjacency is inherited from Fig. 3(c)&(d). This is accomplished by assigning a small number d_0 ($d_0 = 0$ in this implementation) to the cost between two adjacent AM nodes (or SM nodes). As shown in Fig. 8(a), the route from N_i^a to N_{i+1}^a is zerocost, meaning $C(N_i^a, N_{i+1}^a) = d_0$, but the route from N_{i+1}^a to N_i^a is forbidden, meaning $C(N_{i+1}^a, N_i^a) = -1$. This is different from Fig. 8(b) in which the cost between $N_{i+1,j}^s$ and $N_{i+1,j}^s$ is d_0 , i. e., $C(N_{i+1,j}^s, N_{i+1,j}^s) = C(N_{i+1,j}^s, N_{i+1,j}^s) = d_0$.

For both standalone AM and SM operations, reducing non-productive travel is essential, so the cost of the internal air-travel moves of AM or SM is assigned as a relatively large number d_{aa} and d_{ss} , respectively, as shown in Fig. 8(a)&(b). Note that there must have $d_{aa} < d_{as}$ and $d_{ss} < d_{as}$, and in this implementation, $d_{aa} = d_{ss} = 100$ for simplification. This amounts to assigning d_{aa} (d_{ss}) to the undirected edge between N_i^a and N_k^a ($N_{i+1,j}^s$ and $N_{i+1,j}^s$) (see Fig. 8(d)) and is equivalent to the matrix form: $C(N_i^a, N_k^a) = C(N_k^a, N_i^a) = d_{aa}$ and $C(N_{i+1,j}^s, N_{i+1,j}^s) = C(N_{i+1,j}^s, N_{i+1,j}^s) = d_{ss}$.

For the DED process, the cooling strategies, which are based on the part shape and the DED thermal history, could also be integrated as preferences into the unified cost matrix C . For example, for multiple branches of a part, if a cooling break is required after printing N_i^a of a branch, the cost d_{aa} from N_i^a to N_k^a of another branch could be set as a small number such that the inter-branch jump $N_i^a \rightarrow N_k^a$ is more likely to be accomplished. On the other hand, if it is better to stay on the same entity to avoid too much cooling, the cost d_{aa} between entities, i.e., $N_i^a \rightarrow N_k^a$, could be correspondingly increased.

2.4. General weighted digraph for HM sequence planning

The HM constraints and preferences in Sections 2.2 and 2.3 pose the *hard* and *soft* precedence requirements on the HM digraph, respectively. This forms a general weighted digraph for HM in which the directed edges represent the hard constraints while the weights on directed/undirected edges reflect the preferences. The objective is to find a minimum-cost Hamiltonian path that visits each node exactly once while adhering to all the established precedence constraints. In other words, the HM sequence planning is modeled as an SOP (Skinderowicz, 2017), which can be efficiently solved by the Lin-Kernighan-Helsgaun (LKH) heuristic algorithm (Helsgaun, 2017), facilitating a clear decoupling of the problem formulation from the optimization execution.

In order to implement this SOP, the digraphs (Fig. 6(c), Fig. 7(c) and Fig. 8(d)) are integrated into a dense cost matrix C whose element values $C(N_a^*, N_b^*)$ denote the cost from an arbitrary HM node N_a^* to another node

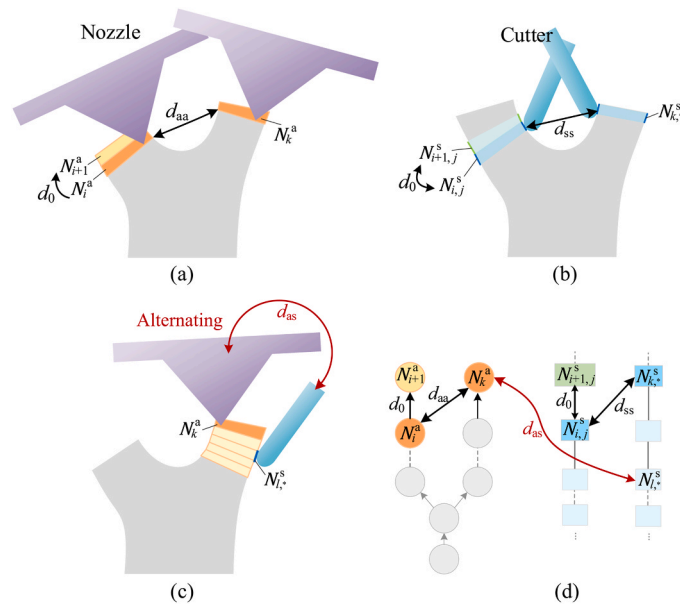


Fig. 8. Preferences in HM: (a) the zero-cost d_0 of AM and its air movement cost d_{aa} ; (b) the zero-cost d_0 of SM and its air movement cost d_{ss} ; (c) the alternation cost d_{as} between AM and SM; (d) the costs from (a)-(c) are assigned to the (un) directed edges of the HM digraph.

N_b^* . Note that $C(N_a^*, N_b^*) \neq C(N_b^*, N_a^*)$ and the hard precedence constraint is represented as $C(N_a^*, N_b^*) = -1$ if the visit from N_a^* to N_b^* is not permitted (N_b^* must be visited before N_a^* but is NOT necessarily adjacent to N_a^*). The soft HM preference costs d_{aa} , d_{ss} , d_{as} and d_0 are placed in the cost matrix C , respectively.

Fig. 9 illustrates a typical cost matrix encoding the HM constraints and preferences, in which the first n_A rows (columns) correspond to the AM nodes while the last n_S rows (columns) the SM nodes. Thus, the total cost matrix $C \in \mathbb{R}^{(n_A+n_S) \times (n_A+n_S)}$ is composed of four functional blocks:

$$C = \begin{bmatrix} C_{11}^{n_A \times n_A} & C_{12}^{n_A \times n_S} \\ C_{21}^{n_S \times n_A} & C_{22}^{n_S \times n_S} \end{bmatrix} \quad (4)$$

Specifically, firstly, the upper-left sub-matrix $C_{11}^{n_A \times n_A}$ reflects the connection between the AM nodes, in which “-1” means the dependency or collision constraints, d_0 denotes the zero-cost AM between adjacent printing layers, and d_{aa} denotes the cost of air movement between the two non-adjacent layers. Then, the upper-right sub-matrix $C_{12}^{n_A \times n_S}$ reflects the collision with the AM nodes when machining the SM nodes (for “-1”), as well as the alternating cost between AM and SM. Next, some of the “-1” elements in the lower-left sub-matrix $C_{21}^{n_S \times n_A}$ represent the natural dependency between AM and SM (a layer can only be machined after it is printed) while others represent the collision with the SM nodes when printing the AM nodes. Finally, the d_0 in the lower-right sub-matrix $C_{22}^{n_S \times n_S}$ denotes the zero-cost between adjacent potential machining paths, d_{ss} denotes the cost of air moment between two non-adjacent machining paths, and “-1” means the inferred collision condition with the SM nodes when machining the SM nodes.

The virtual terminal nodes and isolated nodes for graph traversal. For multi-branch or high-genus structures, there will be multiple starting nodes or leaf nodes, as shown in Fig. 10. To search for a better traversal sequence, two virtual terminal nodes N_T are added: one is for starting the traversal and the other is for ending it. The terminal nodes have a distance of d_{as} to all the HM nodes but d_0 to all the isolated nodes. The isolated nodes N_{iso} are the ones that cannot be machined with any tool directions by a given cutter, which have a distance of d_0 to any other N_{iso} nodes and terminal nodes, but a distance of d_{as} to all the other ordinary HM nodes.

The validity of the HM cost matrix. The digraph represented by the cost matrix C in Eq. (4) should be a directed acyclic graph to ensure the feasibility of operation sequencing. Any cycles within the digraph would imply a logical contradiction in the manufacturing precedence. The presence of such cycles is verified through standard topological sorting algorithms prior to the optimization phase.

3. Property-controllable sequence optimization for continuous multi-axis HM

Section 2 shows that the HM sequence planning problem could be

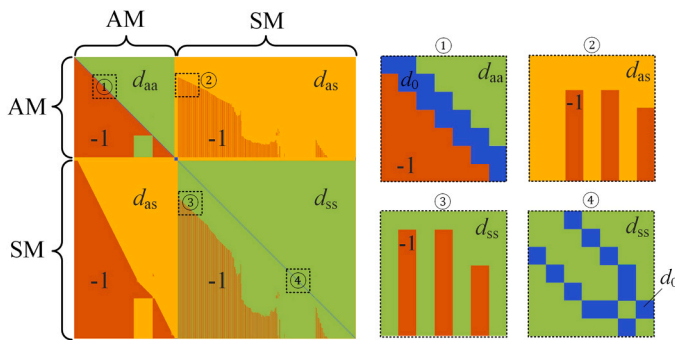


Fig. 9. A typical cost matrix C composed of four blocks: AM-AM, AM-SM, SM-AM, and SM-SM, whose enlarged views are shown on the right (the tee pipe model in Fig. 3).

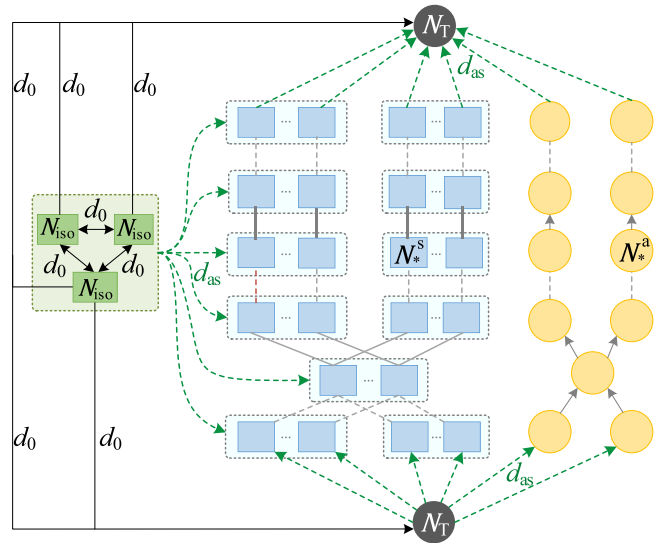


Fig. 10. The virtual terminal nodes N_T and isolated nodes N_{iso} , among them different costs are assigned.

converted into an SOP optimization, assuming that the dependency and collision-free constraints have been identified. However, the nozzle/cutter's collision with the in-process-workpiece (IPW) are highly sensitive to the nozzle/cutter orientations, although their tool paths are prespecified. This section details the determination of tool orientations and evaluation of the corresponding manufacturability (i.e., the printability for AM and machinability for SM, respectively).

3.1. Printability computation

The printability of an AM node N_i^a means it can be printed without any collision between the nozzle assembly and a given AM node N_j^a (or SM node $N_{k,s}^s$), under the dependency requirements. Taking an arbitrary printing point $p \in N_i^a$ for example, the collision check is performed as follows.

3.1.1. Nozzle axis determination for multi-axis DED

After the adaptive slicing (Eisenbarth et al., 2021) in Section 2.1, as shown in Fig. 11(a), for each slice, the commonly used contour parallel method is employed, in which all the boundaries (for non-zero genus slice) of the slice are set as the source for path offsetting. For each point p on a path, to ensure deposition stability, a continuously varied multi-axis nozzle orientation should be assigned by slightly tilting the nozzle axis a at p , towards the face normal direction n_f . The details can be found in the previous work (He et al., 2024a).

At each point p with its nozzle axis a determined, the collision of the nozzle is checked, which is approximated as two truncated cones plus a bounding cylinder, as shown in Fig. 11(b). Specifically, the distance between the nozzle and p (i.e., the standoff distance S) is fixed at 40 mm when printing all the points under the multi-axis scheme. Below the nozzle assembly, there is a virtual cone between the nozzle and the workpiece whose apex coincides with p , and the shape approximates the powder flow. In the implementation, the virtual cone is segmented into two parts: the upper one (with a height of $d_u = 35$ mm) and the lower one (with a height of $d_l = 5$ mm) (Fig. 11(b)). The collision between the upper one and the workpiece is checked (dark gray) since the powder flow should not be blocked, while the collision of the lower one is not checked to leave an allowance for p .

The collision check procedure is accelerated by GPU parallel computing and spatial hierarchical data structures such as the axis-aligned bounding box tree (also applicable to the collision check of the cutter in Section 3.2).

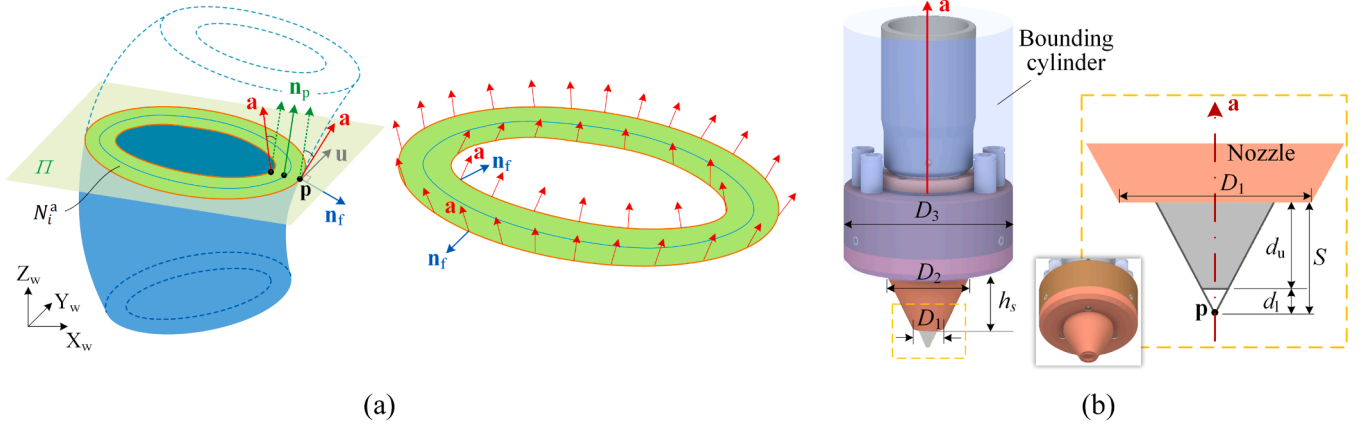


Fig. 11. (a) The continuously varied nozzle orientation $\mathbf{a}$ in multi-axis DED; (b) the nozzle assembly used in the experiment, by which the geometric primitives for collision detection are constructed.

3.1.2. Printability analysis

It is computationally prohibitive to evaluate the printability of N_i^a w.r.t. a potential obstacle AM node N_j^a by performing a collision check at each point $\mathbf{p} \in N_i^a$. In this study, under the proposed planar slicing scheme, the computation is boosted utilizing the AM digraph (Fig. 3(b)). For each AM node N_i^a , its parent AM nodes are searched recursively in the AM digraph and collected into a set $\{N_{pre}^a\}_i$. If a node $N_j^a \in \{N_{pre}^a\}_i$, the precedence constraint $N_j^a \rightarrow N_i^a$ is added and the collision check of N_i^a w.r.t. N_j^a is thus skipped. Also, the collision check is skipped when printing N_j^a w.r.t. N_i^a since the collision obstacle does not exist at all.

After excluding the above AM nodes, only the AM node pairs $\langle N_i^a, N_j^a \rangle$ without a direct or transitive dependency constraints require rigorous collision evaluation. For these pairs, a representative set of points $\mathbf{p} \in N_i^a$ is sampled and the collision condition of the nozzle assembly against N_j^a is verified at each point $\mathbf{p}$.

As discussed in Section 2.2, the printability of N_i^a w.r.t. a potential obstacle SM node $N_{k,*}^s$ can be directly inferred from its printability w.r.t. the corresponding potential obstacle AM node N_k^a , further streamlining the constraint identification process.

3.2. Machinability computation

The machinability of an SM node N_{ij}^s means it can be machined without any collision between the cutter assembly and a given AM node N_k^a (or SM node $N_{k,*}^s$), under the dependency requirements. A node N_{ij}^s is machinable w.r.t. AM node N_k^a if the cutter does not collide with N_k^a when machining all the cutter contact (CC) points $\mathbf{p}_{cc} \in N_{ij}^s$. However, different from printing, in the machining process, the cutter orientations are undetermined and scattered on the Gaussian sphere. Conventionally, reasonable cutter axes can be determined only after the entire HM sequence is obtained, and different cutter axes influence the machinability reversely, thereby the HM sequence, which leads to a "chicken-and-egg" problem. In this paper, a novel concept of accessibility heatmap is proposed to rank each valid cutter axis, based on which the axes along the CC path are optimized considering the axes' smoothness and accessibility ranking, followed by an overall machinability evaluation of an SM node with fixed cutter axes.

3.2.1. Accessibility heatmap construction

In the previous work (Chen et al., 2020), cutter directions are sampled on a Gaussian sphere but they are not finalized until the entire sequence planning is completed. Thus, the collision condition of the CC point $\mathbf{p}_{cc} \in N_{ij}^s$ with the obstacle node N_k^a (discussed in Section 2.2)

remains uncertain, hindering the formulation of the total cost matrix for SOP (Section 2.4). Thus, the key to this problem is the determination of the cutter axes at CC points. To solve this issue, in this paper, the accessibility heatmap is proposed to reflect the optimality of each tool axis at each CC point that is then optimized considering the axial smoothness.

A key observation is that the cutter axis $\mathbf{a}$ at each CC point can be pre-assigned and optimized such that the cutter with axis $\mathbf{a}$ collides with the printed region as late as possible although the cutter may collide with the final part. This is based on the fact that the machinability of each CC point $\mathbf{p}_{cc}$ is only affected by the AM sequence, while the SM sequence will not influence the accessibility of AM nodes, assuming the as-built allowance is small (the detailed explanation is given at the end of Section 3.2.2). Specifically, firstly, as shown in Fig. 12(a), at each CC point $\mathbf{p}_{cc} \in N_{ij}^s$, many directions $\mathbf{a}$ are uniformly sampled as the cutter axis on a Gaussian sphere satisfying the following constraint

$$\mathbf{a} \cdot \mathbf{n}_f > 0 \text{ and } \mathbf{a}_z > -\sin \alpha_{lim} \quad (5)$$

where $\mathbf{n}_f$ denotes the surface normal at $\mathbf{p}_{cc}$; $\mathbf{a}_z = \mathbf{a} \cdot [0, 0, 1]^T$ denotes the Z-component of $\mathbf{a}$; α_{lim} denotes the maximum allowed angle of the axis $\mathbf{a}$ below the horizontal plane in the workpiece coordinate system.

As shown in Fig. 13, for each direction $\mathbf{a}$ on the Gaussian sphere centered at $\mathbf{p}_{cc}$, the collision between the cutter (with the axis $\mathbf{a}$) and each AM node (layer) and the table/fixture is checked, in which the cutter is approximated by a sphere, a cylinder, and a truncated cone, as shown in Fig. 12(b). The AM nodes are arranged into a sequence $\{N_0^a, N_1^a, \dots, N_m^a, \dots, N_{n_A-1}^a\}$ which is directly obtained from a layer-level pure AM sequence planning task (Lau et al., 2023; Lensgraf and Mettu, 2016; Ma et al., 2023; Yuan et al., 2022; Zhong et al., 2023a), considering the dependency and collision constraints in AM (Section 2.2). Fortunately, this problem is exactly a sub-SOP task where the cost matrix is the top left sub-matrix $\mathbf{C}_{11}^{n_A \times n_A}$ of Eq. (4) (see Fig. 9).

Now, the feasibility (accessibility) of each direction $\mathbf{a}$ sampled as shown in Fig. 12(a) is evaluated at the CC point $\mathbf{p}_{cc}$, denoted as $f(\mathbf{a}; \mathbf{p}_{cc})$. As illustrated in Fig. 13(b), if the cutter $\mathcal{T}_s(\mathbf{p}_{cc}, \mathbf{a})$ with $\mathbf{p}_{cc}$ and $\mathbf{a}$ collides with the table or other existing fixture $\mathcal{F}$, $f(\mathbf{a}; \mathbf{p}_{cc}) = -1$; if $\mathcal{T}_s(\mathbf{p}_{cc}, \mathbf{a})$ does not collide with the design model Ω , the accessibility is set as the largest index n_A – the total number of AM nodes; otherwise, each AM node is visited one-by-one from the AM sequence $\{N_0^a, N_1^a, \dots, N_m^a, \dots, N_{n_A-1}^a\}$ (Fig. 13(a)), and the first AM node that collides with $\mathcal{T}_s(\mathbf{p}_{cc}, \mathbf{a})$ is obtained, i.e., the accessibility $f(\mathbf{a}; \mathbf{p}_{cc})$ is set as the smallest index of the layer which obstructs the cutter. The above criteria are

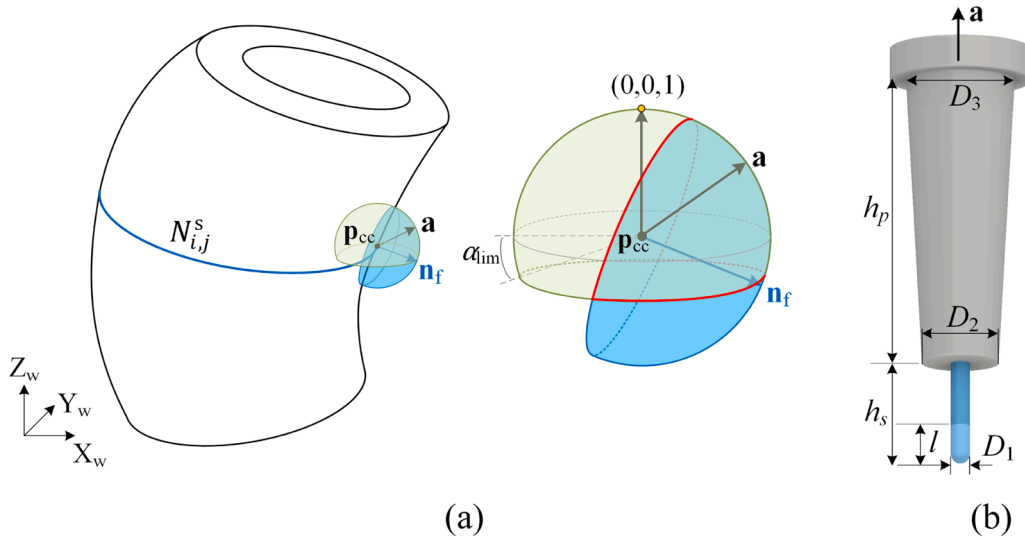


Fig. 12. (a) The tool axes at CC point $\mathbf{p}_{cc}$ are sampled on the Gaussian sphere; (b) the cutter assembly used in the experiment, by which the geometric primitives for collision detection are constructed.

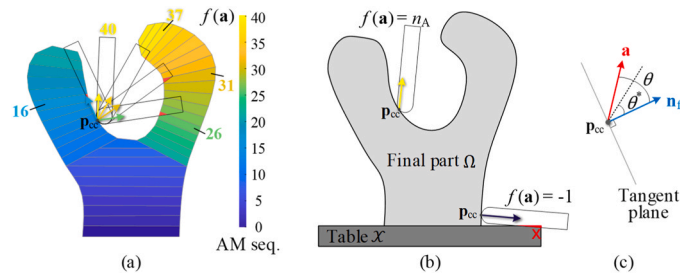


Fig. 13. (a) The feasibility $f(\mathbf{a})$ of direction $\mathbf{a}$ is an integer representing the smallest index of the ordered AM layers with which the cutter collides (shown in red); (b) $f(\mathbf{a}) = -1$ if the cutter collides with the table/fixture; $f(\mathbf{a}) = n_A$ (the largest index) if the cutter is totally collision-free; (c) the remainder representing the preference of cutter orientations which means $\mathbf{a}$ is expected to form an angle of θ^* with the face normal (2D illustration).

described as follows:

$$f(\mathbf{a}; \mathbf{p}_{cc}) := \begin{cases} -1, & \mathcal{T}_s(\mathbf{p}_{cc}, \mathbf{a}) \cap \mathcal{X} \neq \emptyset \\ n_A, & \mathcal{T}_s(\mathbf{p}_{cc}, \mathbf{a}) \cap \Omega = \emptyset \\ \argmin_{m=0,1,\dots,n_A-1} \{N_m^a | \mathcal{T}_s(\mathbf{p}_{cc}, \mathbf{a}) \cap N_m^a \neq \emptyset\}, & \text{otherwise} \end{cases} \quad (6)$$

Eq. (6) shows that the accessibility is an integer representing the smallest index of the ordered layers with which the cutter collides. Nevertheless, two directions with a same accessibility index are ranked through a remainder $\mathcal{R}(\mathbf{a}; \mathbf{p}_{cc})$ which is a real number between 0 and 1, reflecting the closeness of the axis $\mathbf{a}$ to the cone with a taper angle of θ^* centered along the face normal $\mathbf{n}_f$, as shown in Fig. 13(c). Also, this $\mathcal{R}(\mathbf{a}; \mathbf{p}_{cc})$ is further modulated by the angle to $[0, 0, 1]^T$, i.e.,

$$\mathcal{R}(\mathbf{a}; \mathbf{p}_{cc}) = \left(1 - \frac{|\theta - \theta^*|}{\max(\theta^*, \pi/2 - \theta^*)}\right) \cdot \left(1 - \frac{\text{acos} \mathbf{a}_z}{\pi/2 + \alpha_{lim}}\right) \quad (7)$$

where $\theta = \text{acos}(\mathbf{a} \cdot \mathbf{n}_f)$ and the $\mathcal{R}(\mathbf{a}; \mathbf{p}_{cc})$ in Eq. (7) represents a preference for aligning the cutter axis $\mathbf{a}$ at an angle θ^* (say 30°) relative to $\mathbf{n}_f$, while striving to keep it as close as possible to $[0, 0, 1]^T$. It is guaranteed that $\mathcal{R}(\mathbf{a}; \mathbf{p}_{cc}) \in [0, 1]$ which is added back to $f(\mathbf{a}; \mathbf{p}_{cc})$ as a fractional part such that the accessibility $f(\mathbf{a}; \mathbf{p}_{cc})$ is a real number and the integer part of $f(\mathbf{a}; \mathbf{p}_{cc})$ is still preserved:

$$f(\mathbf{a}; \mathbf{p}_{cc}) \leftarrow f(\mathbf{a}; \mathbf{p}_{cc}) + \mathcal{R}(\mathbf{a}; \mathbf{p}_{cc}), f(\mathbf{a}; \mathbf{p}_{cc}) \neq -1 \quad (8)$$

With the feasibility $f(\mathbf{a}; \mathbf{p}_{cc})$ obtained, a spherical accessibility heatmap is constructed on a Gaussian sphere for each CC point, as shown in Fig. 14(a) (where the points' intervals are exaggerated for visualization). The color in the heatmap represents the accessibility $f(\mathbf{a}; \mathbf{p}_{cc})$ of each cutter axis. Fig. 14(a) shows two SM nodes $N_{i,0}^s$ and $N_{i,1}^s$ on which CC points exhibit different accessibilities and the distributions of feasibility indices on some Gaussian spheres are entirely discontinuous. Generally, the CC points outside the exemplary tube structure in Fig. 3(a) (belonging to $N_{i,0}^s$) exhibit superior accessibility compared to their internal counterparts (belonging to $N_{i,1}^s$). Specifically, the point labeled ① possesses a broad range of collision-free directions in bright yellow, although certain directions result in interference with the final AM layers of the model. Conversely, the point labeled ② is obstructed so severely that no direction is completely collision-free after the whole part Ω is finished. Instead, the directions towards the left window of the tube show better accessibility (lighter colors). The inner points by ③ have directions with distinct performances – the ones directed inwards have small indices, while the ones pointing to the right window of the tube are completely collision-free with Ω . These complex, spatially varying accessibilities underscore the necessity of robust tool-axis optimization along the toolpath, as detailed in the subsequent section.

3.2.2. Cutter axis determination considering smoothness and accessibility

The prepotency of each direction for cutter orientation planning is quantified by the accessibility heatmap, i.e., at each CC point $\mathbf{p}_{cc}$, an optimal direction must be selected from the corresponding heatmap. Thus, to delay the potential collision of $\mathbf{p}_{cc}$ as much as possible, one intuitive method is to select the direction with the largest $f(\mathbf{a}; \mathbf{p}_{cc})$, which, however, may generate rugged tool axes along the CC path. In this paper, a smoothness term along the CC path is introduced and a graph-based search algorithm is employed for generating smooth and feasible tool axes.

Fig. 15 shows a multilayer graph in which each column represents a CC point (from CC_1 to CC_n) and each row corresponds to a sampled direction on the Gaussian sphere. Two nodes $\mathbf{a}_j^i$ and $\mathbf{a}_k^{i+1}$ from adjacent CC points CC_i and CC_{i+1} are connected by an edge whose weight $w(\mathbf{a}_j^i, \mathbf{a}_k^{i+1})$ balances the angle formed by $\langle \mathbf{a}_j^i, \mathbf{a}_k^{i+1} \rangle$ and the feasibilities of $\mathbf{a}_j^i$ and $\mathbf{a}_k^{i+1}$:

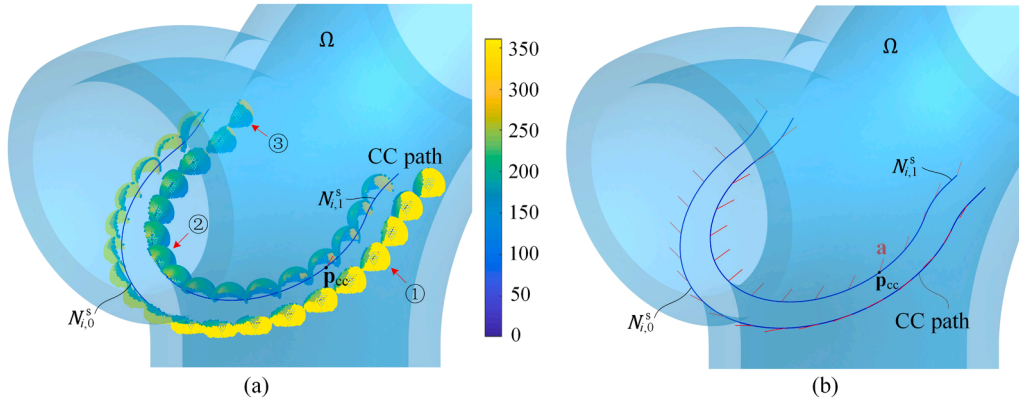


Fig. 14. (a) The accessibility heatmap at each CC point p_{cc} along CC paths where the color represents the accessibility of each cutter axis; (b) the optimized cutter orientations along the CC path, considering the accessibility and the smoothness of tool axes.

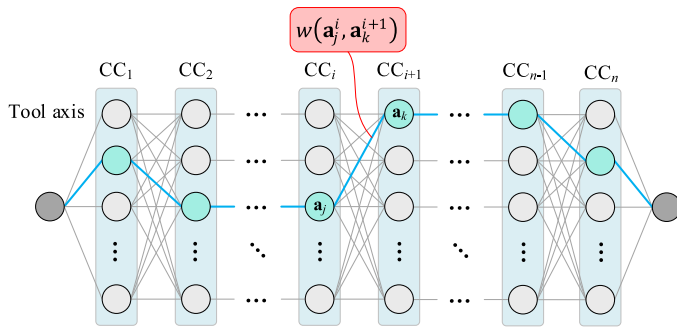


Fig. 15. A graph-based search algorithm for cutter orientations along a CC path, in which both the smoothness and accessibility are considered and optimized.

$$w(a_j^i, a_k^{i+1}) = (1 - \lambda) \left(\frac{1 - a_j^i a_k^{i+1}}{2} \right) + \lambda \left(1 - \frac{f(a_j^i) + f(a_k^{i+1})}{2} / n_A \right), \quad \lambda \in [0, 1] \quad (9)$$

where n_A denotes the total number of AM nodes, and $\lambda \in [0, 1]$ is a weighting coefficient that regulates the trade-off between the smoothness (small λ) and feasibility (large λ). Finding a smooth and feasible tool axes sequence along the CC path amounts to searching for the minimum-cost path through this graph, which can be efficiently solved via the Dijkstra's algorithm (Dijkstra, 1959). To optimize and boost the accessibility at the endpoints, two virtual nodes are introduced at the beginning and the end for this graph-based search (in dark gray), respectively. The optimized cutter orientations along CC paths are illustrated in Fig. 14(b), and each tool axis is likely to have high feasibility and smoothness with adjacent ones. After the cutter axis a on each CC point has been determined, the global feasibility of an SM node N_{ij}^s is defined by the minimal feasibility index encountered along the path:

$$f(N_{ij}^s) = \min_{p_{cc} \in N_{ij}^s} f(a; p_{cc}) \quad (10)$$

To improve the computing efficiency, the collision detection in each direction is accelerated using GPU-based parallel computing. Furthermore, the sampling resolution on the Gaussian sphere is iteratively refined in conjunction with the graph-based optimization to ensure high-fidelity tool-axis generation.

Remarks: The implementation of the accessibility heatmap is predicated on the ordering invariance principle observed in this study.

Specifically, the order of the AM nodes in the final HM sequence $N_i^a \dots N_{s,*}^s \dots N_j^a \dots N_{s,*}^s \dots N_k^a$ (Fig. 16(b)) after solving the full-SOP with a total cost matrix $C^{(n_A+n_S) \times (n_A+n_S)}$, remains identical to the order derived from the pure AM sequence (Fig. 16(a)) after solving sub-SOP with sub-cost matrix $C_{11}^{n_A \times n_A}$, provided the stochastic parameters of the LKH algorithm are held constant. This is true because inserting the SM nodes will not add, delete, or change the connections among the AM nodes themselves, or break the symmetry of 2 AM nodes connected by undirected edges, as long as the symmetry of the connection with SM nodes is preserved.

As shown in Fig. 16(c), in pure AM planning, the hard constraint $N_i^a \rightarrow N_j^a$ must be guaranteed, which is also naturally satisfied after incorporating the SM nodes since the above precedence requirement is also input to the full-SOP algorithm. In Fig. 16(d), N_i^a and N_j^a are connected by an undirected edge, indicating a lack of strict precedence constraint. In this case, inserting the SM node group on the right still introduces a set of undirected edges that are further abstracted into one undirected edge with the summed symmetric weight $\sum d_s$, which has no effect on the sequence preference between N_i^a and N_j^a . If the pure AM sequence planning result visits N_i^a before N_j^a , after the HM sequence planning, N_i^a will be placed before N_j^a as well. This finding indicates that to solve an HM sequence planning problem, a sub-problem needs to be

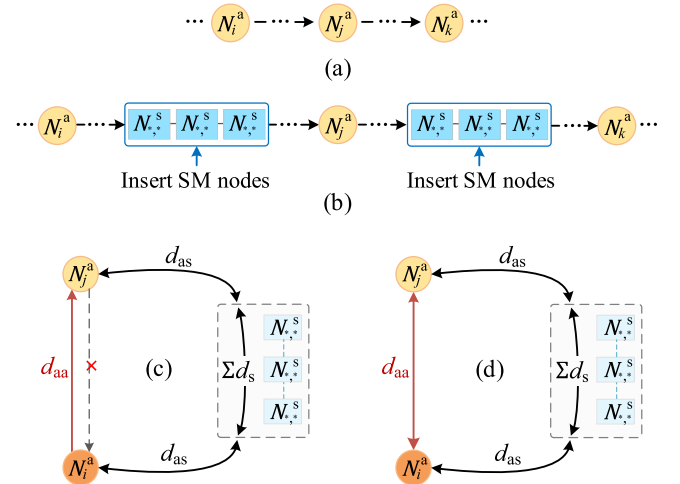


Fig. 16. The order of AM nodes in pure AM sequence planning (a) is identical to the one after inserting the SM nodes by solving the HM sequence planning (b). This is because inserting SM nodes will not change the connection among the AM nodes (c), or break the symmetry of any 2 AM nodes connected by undirected edges (d).

solved for AM sequence planning beforehand.

3.2.3. Machinability analysis

Given the cutter axis at each CC point $\mathbf{p}_{cc}$, the local machinability of $\mathbf{p}_{cc}$ w.r.t. N_k^a is defined as whether this cutter collides with the potential obstacle AM node N_k^a . Consequently, the machinability of an SM node $N_{i,*}^s$ w.r.t. N_k^a is defined as whether all the points $\mathbf{p}_{cc} \in N_{i,*}^s$ along the path are machinable without collision with N_k^a . In order to construct the sub-matrices $\mathbf{C}_{12}^{n_A \times n_S}$ and $\mathbf{C}_{21}^{n_S \times n_A}$ in Eq. (4), the machinability of each S-A node pair $\langle N_{i,j}^s, N_k^a \rangle$ must be tested, i.e., whether $N_{i,j}^s$ can be machined without collision with N_k^a given the tool axes along the CC path, which, however, is time-consuming. Fortunately, this could be accelerated using the dependency information of $\mathbf{C}_{11}^{n_A \times n_A}$, as described below.

First, the machinability of each $\mathbf{p}_{cc} \in N_{i,*}^s$ is tested, and if one or more CC points are found to have no valid axis or the feasibility $f(\mathbf{a}; \mathbf{p}_{cc}) = -1$, $N_{i,*}^s$ becomes totally inaccessible as it collides with all the AM nodes. Otherwise, the first index $k = \lfloor f(\mathbf{a}; \mathbf{p}_{cc}) \rfloor$ in the pure AM sequence is obtained with which the cutter $\mathcal{T}_s(\mathbf{p}_{cc}, \mathbf{a})$ collides, i.e., $N_k^a \rightarrow N_{i,*}^s$. Next, starting from k , each AM node N_k^a ($k \geq k$) in the pure AM sequence is traversed and if $\mathbf{C}_{11}(N_k^a, N_k^a) = -1$, which means $N_k^a \rightarrow N_k^a$, it can be inferred that $N_k^a \rightarrow N_{i,*}^s$ and $\mathbf{C}_{12}(N_k^a, N_{i,*}^s) = -1$ without further calculation. Explicit collision detection is only performed for AM nodes that do not have existing precedence constraints to N_k^a and if the collision occurs, $\mathbf{C}_{12}(N_k^a, N_{i,*}^s)$ is set to -1 , which significantly reduces the total number of geometric intersection tests.

3.3. Property-controllable enabling

The proposed graph-based HM sequence planning method is highly flexible and facilitates the embedding of various physical properties to regulate the behavior at the interface between AM and SM, and more importantly, these properties can be individually superposed or integrated into the total cost matrix $\mathbf{C}^{(n_A+n_S) \times (n_A+n_S)}$. In this paper, three functional embeddings are demonstrated: adaptive remelting avoidance, interface regulation, and machining sequence optimization, and other physical preferences or constraints for integration are also allowed.

3.3.1. Adaptive remelting avoidance

The deposition of each AM node N_i^a will generate significant thermal load, which may remelt the machined layers below it and degrade the surface quality, as shown in Fig. 17(a). To mitigate this, a few SM nodes beneath N_i^a need to be machined after N_i^a is deposited (Chen et al., 2025; Nishiyama et al., 2023). Thus, for each AM node N_i^a , its predecessors are recursively traversed within the AM diagram (Fig. 3(b)) to a search depth n_{rmt} , representing the number of layers affected by the remelting. This traversal identifies a set of affected AM layers

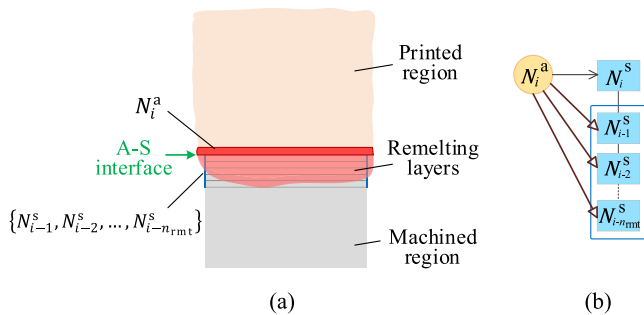


Fig. 17. The A-S constraint for remelting avoidance: each AM node N_i^a must be printed before its corresponding remelting layers $\{N_{i-1}^s, N_{i-2}^s, \dots, N_{i-n_rmt}^s\}$ are machined.

$\{N_{i-1}^a, N_{i-2}^a, \dots, N_{i-n_rmt}^a\}$ whose associated SM nodes can then be collected to infer the affected SM nodes $\{N_{i-1}^s, N_{i-2}^s, \dots, N_{i-n_rmt}^s\}$, according to the definition of HM unit (Fig. 4(c)).

The remelting avoidance constraint requires that the SM nodes $\{N_{i-1}^s, N_{i-2}^s, \dots, N_{i-n_rmt}^s\}$ must be visited after N_i^a , which amounts to, apart from the A-S dependency $N_i^a \rightarrow N_i^s$, adding additional dependency constraints $N_i^a \rightarrow \{N_{i-1}^s, N_{i-2}^s, \dots, N_{i-n_rmt}^s\}$, as shown in Fig. 17(b). Further, considering that the remelting effect is related to the heat dissipation efficiency which can be roughly estimated by the area of the cross-section being printed, i.e., $n_{rmt}(\mathcal{A}(N_i^a))$, more flexibility can be introduced by varying the number of affected layers n_{rmt} with different positions in the model Ω , which can be achieved by adapting the search depth from N_i^a to different locations of N_i^a itself.

The remelting-avoidance constraint may restrict the solution space for HM sequence planning and even cause an interlock issue where no solution exists at all. To resolve this, another adaptive remelting avoidance scheme is to select a proper n_{rmt} for each AM node in a manner that preempts A-S interlocks (Fig. 20(d)&(e)). Specifically, as shown in Fig. 20(d), a conflict often arises from two competing manufacturing requirements. On the one hand, the above remelting avoidance requires that the SM node $N_{i-n_rmt,*}^s$ be performed after the AM node N_i^a has been deposited, i.e., $N_i^a \rightarrow N_{i-n_rmt,*}^s$. On the other hand, when machining the SM node $N_{i-n_rmt,*}^s$, the tool accessibility region becomes so small that the cutter with the optimized cutter axis obtained from Section 3.2.2 would collide with the deposited N_i^a , which means $N_{i-n_rmt,*}^s$ should be machined before N_i^a , as shown in Fig. 20(e). This causes the A-S interlock issue, which has to be resolved by the proposed adaptive determination method for the number of remelting layers n_{rmt} .

In order to determine the adaptive remelting layer numbers for each AM node of interest N_i^a , all the AM nodes N_r^a before N_i^a are collected whose corresponding SM nodes collide with N_i^a , i.e., $\lfloor f(N_r^a) \rfloor = N_i^a$. The adaptive remelting layer number $n_{rmt}(N_i^a)$ of N_i^a is then set as the minimal search depth from N_i^a to N_r^a if it is less than the preset number n_{rmt}^*

$$n_{rmt}(N_i^a) = \min \left\{ \min_{r \in \{i_{pre}\}} \mathcal{D}(N_i^a, N_r^a), n_{rmt}^* \right\} \quad (11)$$

$$\text{s.t. } \lfloor f(N_r^a) \rfloor = N_i^a, r \in \{i_{pre}\}$$

where $\mathcal{D}(N_i^a, N_r^a)$ denotes the search depth from N_i^a to N_r^a and $\{i_{pre}\}$ are the previously deposited layers of N_i^a . This could be implemented by adding an exit condition of downward searching from N_i^a – if $\lfloor f(N_r^a) \rfloor = N_i^a$, the search terminates.

3.3.2. A-S interface regulation

The SOP task defined in Section 2 formulates the HM sequence planning problem under strict collision avoidance constraints, in which the A-S interface determines the *highest* IPW that can be built, considering the A-S alternation minimization objective. For instance, as shown in Fig. 18(a), the yellow line $\mathcal{R}$ is the highest one that obstructs the cutter accessibility of the CC points $\mathbf{p}_{cc}$ located at the bottom of the inwall. In certain instances, however, based on specific physical property requirements, the region near $\mathcal{R}$ may be designated as a restricted zone. Consequently, the interface position must be adjusted to facilitate property-dependent HM sequence optimization.

In this paper, the forbiddance map $\mathcal{F}(\mathbf{p}) \in (0, 1]$ is introduced to quantify the spatial resistance against establishing an A-S interface at point $\mathbf{p} \in \Omega$, which is preassigned by users (the colormap in Fig. 18(b)). The map $\mathcal{F}(\mathbf{p})$ enables certain physical properties to control the positions of A-S interfaces by modulating the weights between AM and SM nodes in the sub-matrices $\mathbf{C}_{12}^{n_A \times n_S}$ and $\mathbf{C}_{21}^{n_S \times n_A}$ of Fig. 18(d) (different from Fig. 9). The distance cost between the AM node N_i^a and an arbitrary non-

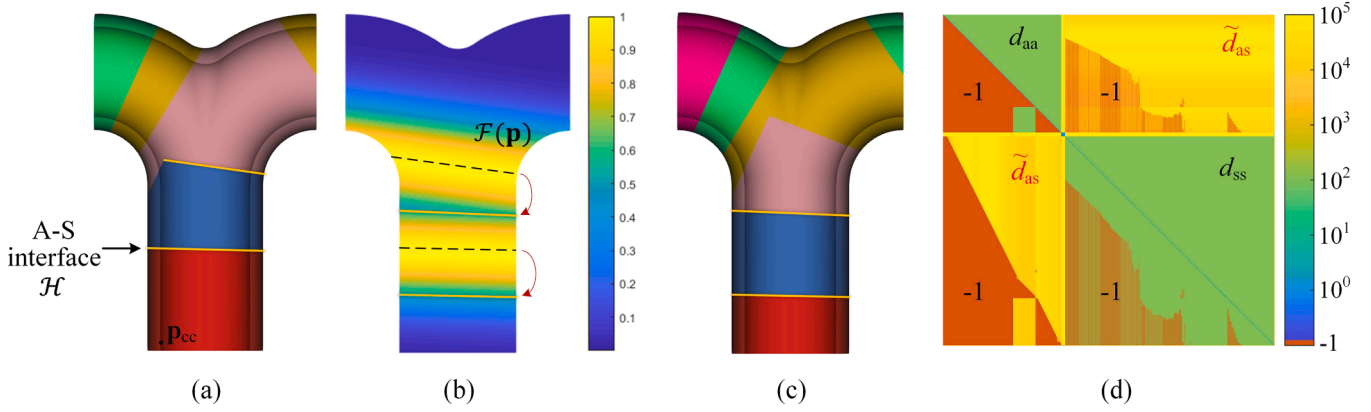


Fig. 18. The property-enabled sequence planning of HM moves the A-S interfaces from the positions in (a) to the ones in (c), according to the forbiddance map $\mathcal{F}(\mathbf{p})$ in (b); (d) the property-enabled cost matrix is different from Fig. 9, in which the varied values between AM and SM represent the forbiddance extent.

isolated node $\forall N_j^s$ is denoted as $\tilde{d}_{as}(N_i^a, \forall N_j^s)$, which is modified from d_{as} by

$$\tilde{d}_{as}(N_i^a, \forall N_j^s) = d_{aa} + \mathcal{F}(\mathbf{p} | \mathbf{p} \in N_i^a) (d_{as} - d_{aa}) \quad (12)$$

where $\mathcal{F}(\mathbf{p} | \mathbf{p} \in N_i^a) = \frac{1}{\mathcal{N}(\mathbf{p})} \sum_{\mathbf{p} \in N_i^a} \mathcal{F}(\mathbf{p})$ denotes the mean value of $\mathcal{F}(\mathbf{p})$ and $\mathcal{N}(\mathbf{p})$ is the number of points of N_i^a . These varied costs between AM and SM can reflect the varied difficulty for A-S alternation – much more difficulties are introduced near the forbidden zone while the alternating costs are relaxed far from the forbidden zone. From Eq. (12), it is guaranteed that $\tilde{d}_{as} > d_{aa}$, indicating that minimizing the number of alternations is still prioritized in the property-controllable scheme. Fortunately, the symmetry of the cost matrix is preserved under varied distances, so the order preservation property of AM nodes in Fig. 16 still holds.

The solution to the SOP with the modified cost matrix elements in Eq. (12) moves the A-S interfaces downward from the original positions (dashed line) to the new ones in orange, as shown in Fig. 18(b), which precisely bypasses the forbidden zone in yellow. Nevertheless, the new HM sequence shortens the AM component (e.g., the red one in Fig. 18(c)) which yields a sub-optimal solution, indicating that the physical requirements are taken care of at the expense of increasing the number of A-S alternations, i.e., from 5 to 6. Fortunately, the A-S interfaces far from the forbidden zone (the blue region in Fig. 18(b)) are little affected – their positions remain nearly identical between Fig. 18(a)&(c). From the computational aspect, when solving the SOP with continuously varied weights $\tilde{d}_{as}$, the HM sequence can be initialized using the one obtained from the constant d_{as} in Section 2.4, which would boost the

computational efficiency.

3.3.3. Machining sequence optimization

After the AM process, the as-built shape (Fig. 19(a)) is typically oversized relative to the design model to provide a machining allowance. This surplus material necessitates careful planning of the cutting sequence between successive SM nodes. However, in the construction of the cost matrix in Sections 2.3 and 2.4, the order between any two adjacent SM nodes is not considered, so the cost on the corresponding undirected edge is d_0 , which makes the machining direction uncertain. As shown in Fig. 19(b), if using the conventional order (the bottom-up direction) where N_{ij}^s is machined before $N_{i+1,j}^s$, the cutter is more likely to collide with the unremoved material left on $N_{i+1,j}^s$. On the contrary, if the adjacent SM nodes are machined in the reverse order of their AM sequence, i.e., $N_{i+1,j}^s \rightarrow N_{ij}^s$, the collision risk is much more alleviated, as shown in Fig. 19(c). Therefore, after a group of SM nodes has been obtained from the HM sequence planning, a sub-SM digraph is constructed from the original SM digraph in Fig. 3(c). Then, the leaf SM nodes of the sub-digraph are identified, and a depth-first search is performed backward till the root node of the sub-digraph. This procedure repeats again until all the nodes of the sub-digraph are visited. This allowance-conscious sequence constraint can also be incorporated into $\mathbf{C}_{22}^{n_s \times n_s}$ in Eq. (4) by assigning the cost from N_{ij}^s to $N_{i+1,j}^s$ as d_{ss} . This naturally forms a feasible machining order within this alternating machining process for alleviating the collision risk. Beyond collision avoidance, this top-down strategy offers the distinct advantage of preserving the structural stiffness of the IPW for as long as possible, which is critical for minimizing machining-induced vibrations and deformations in thin-walled components.

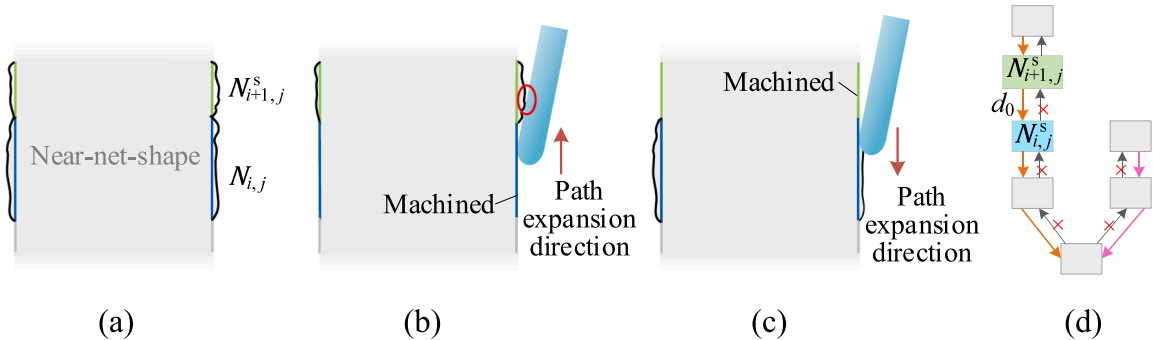


Fig. 19. Machining sequence considering the unremoved material (allowance): (a) the near-net-shape after AM is composed of N_{ij}^s and $N_{i+1,j}^s$; (b) the cutter is likely to collide with $N_{i+1,j}^s$ if N_{ij}^s is machined first; (c) the collision is avoided by reversing the AM order of N_{ij}^s and $N_{i+1,j}^s$, which is represented by the directed edge in (d) rather than an undirected edge.

3.3.4. The interlock issue between HM nodes

Compared with standalone AM or SM sequence planning, the HM has brought much more complexity to the coupled constraints between HM nodes. Further, the physical property-enabled sequence planning introduced extra constraints, which may cause three types of interlock issues between HM nodes: ① A-A interlock, ② S-S interlock and ③ A-S interlock, as discussed below.

The first type is A-A interlock, which means the nozzle not only collides with N_j^a when printing N_i^a but also collides with N_i^a when printing N_j^a as well, as shown in Fig. 20(a), which can be represented by a cycle between N_i^a and N_j^a in the digraph of Fig. 20(c). The A-A interlock usually occurs at multi-branch or high-genus structures. To resolve the A-A interlock, the nozzle orientation can be adjusted, or the slicing parameters/method can be modified (Eisenbarth et al., 2021) which, however, might influence the DED output geometry.

The second type is the S-S interlock which is shown in Fig. 20(b). When machining SM node $N_{i,k}^s$, the cutter collides with the AM node N_j^a , indicating that $N_{i,k}^s \rightarrow N_j^a$. Due to the natural A-S dependency in Fig. 4(c), $N_j^a \rightarrow N_{j,l}^s$ is held, from which $N_{i,k}^s \rightarrow N_{j,l}^s$ is inferred. On the other hand, when machining SM node $N_{j,l}^s$, the cutter collides with the AM node N_i^a , which leads to $N_{j,l}^s \rightarrow N_{i,k}^s$. Thus, the S-S interlock is formed, i.e., $N_{i,k}^s \leftrightarrow N_{j,l}^s$, which is also represented as a cycle in Fig. 20(c). To resolve an S-S interlock, one could use a smaller cutter, while a systematic solution remains to be worked out.

The cause of the third type of A-S interlock has been discussed in Section 3.3.1, and to resolve it, the number of remelting layers can be reduced and the adaptive remelting-avoidance scheme presented in Section 3.3.1 can be employed.

3.4. Path planning for printing and machining

Printing sequence and path planning. The printing sequence within each AM group is inherently determined by the optimized AM node sequence. The specific toolpath generation and parameter adaptation strategies employed for these nodes are detailed in the [supplementary material](#) and the previous work (He et al., 2024a).

Adaptive step-over distance of machining path. The machining path intervals, i.e., the local step-over distances $\delta t(\mathbf{p})$ at point $\mathbf{p}$, are constrained by the layer thickness range $[\delta t_{\min}, \delta t_{\max}]$, as shown in Fig. 21(a). The local step-over distances between CC paths will affect the machined cusp height. For a ball-end cutter with a radius of r_c (the curvature is κ_c), the local cusp height $\mathcal{H}_i^{i+1}(\mathbf{p})$ between two adjacent CC paths $N_{i,j}^s$ and $N_{i+1,j}^s$ is estimated as

$$\mathcal{H}_i^{i+1}(\mathbf{p}) = \frac{\kappa_s(\mathbf{p}, \mathbf{n}_f \times \mathbf{u}) + \kappa_c}{8} \delta t(\mathbf{p})^2 \quad (13)$$

where $\mathbf{u}$ is the feed direction at $\mathbf{p}$, and $\kappa_s(\mathbf{p}, \mathbf{n}_f \times \mathbf{u})$ denotes the signed normal curvature of the surface along the step-over direction $\mathbf{n}_f \times \mathbf{u}$ which is precisely the build direction in AM. If $\max_{\mathbf{p}} \mathcal{H}_i^{i+1}(\mathbf{p}) > \mathcal{H}^*$, an intermediate CC path is inserted as SM node $N_{i,j}^s$ between $N_{i,j}^s$ and $N_{i+1,j}^s$, as shown in Fig. 21(a). Topologically, this requires removing the existing edge $N_{i,j}^s - N_{i+1,j}^s$ in the graph (Fig. 21(b)) and inserting two new edges $N_{i,j}^s - N_{i,j}^s$ and $N_{i,j}^s - N_{i+1,j}^s$. Also, the dependency constraint of SM on AM, i.e., $N_i^a \rightarrow N_{i,j}^s$ should be added. The CC path of $N_{i,j}^s$ can be calculated by intersecting the model Ω with the bisector plane Π_{bi} between slicing planes Π_i and Π_{i+1} , i.e.,

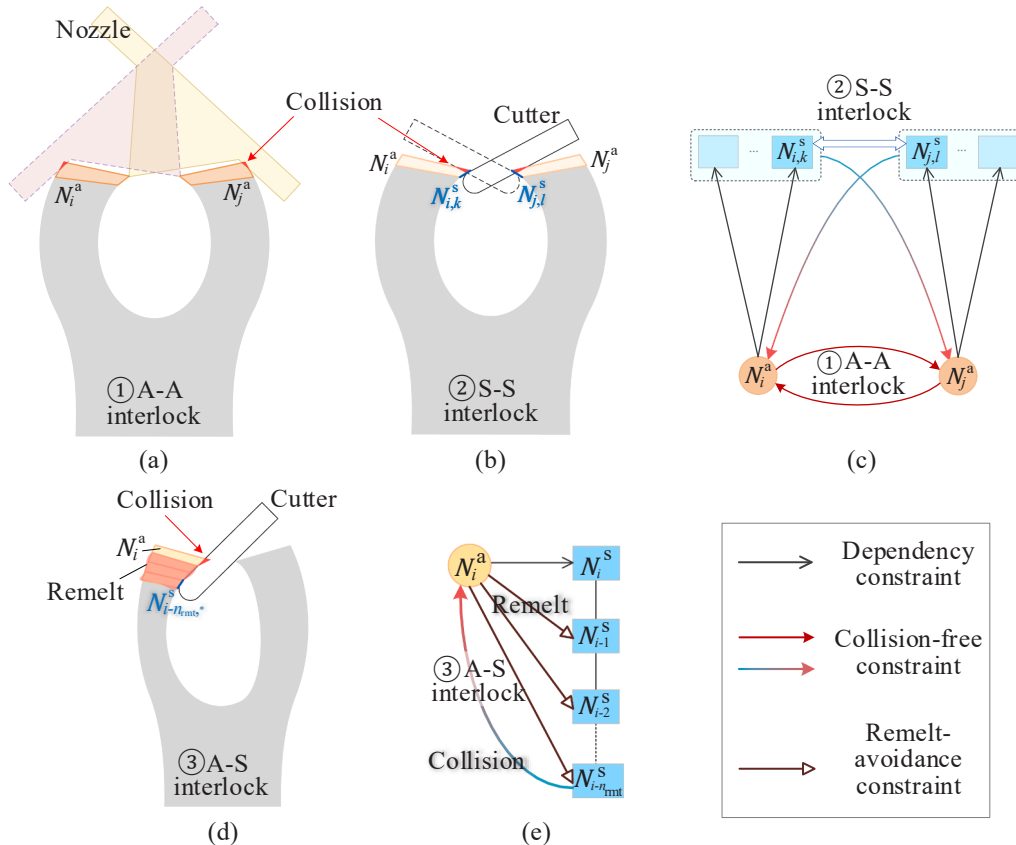


Fig. 20. The interlock issue between HM nodes: the A-A interlock (a) and S-S interlock (b) are abstracted in (c); the A-S interlock (d) is abstracted in (e).

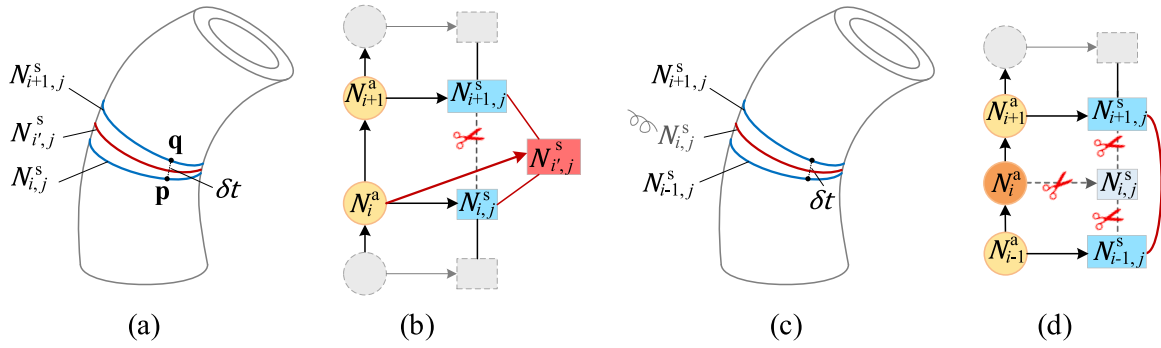


Fig. 21. Adaptive step-over distance of cutting paths: (a) insert a CC path $N_{i,j}^s$ to satisfy the cusp-height requirement; (b) the graph operation of (a); (c) delete a CC path to avoid redundant cutting; (d) the graph operation of (c).

$$\Pi_{bi} : \left(\mathbf{p} - \begin{bmatrix} \mathbf{n}_i^T \\ \mathbf{n}_{i+1}^T \\ (\mathbf{n}_i \times \mathbf{n}_{i+1})^T \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{n}_i^T \hat{\mathbf{p}}_i \\ \mathbf{n}_{i+1}^T \hat{\mathbf{p}}_{i+1} \\ (\mathbf{n}_i \times \mathbf{n}_{i+1})^T \hat{\mathbf{p}}_i \end{bmatrix} \right) \cdot (\mathbf{n}_i + \mathbf{n}_{i+1}) = 0 \quad (14)$$

where $\mathbf{n}_i$, $\hat{\mathbf{p}}_i$, $\mathbf{n}_{i+1}$, and $\hat{\mathbf{p}}_{i+1}$ denote the normals and anchor points of Π_i and Π_{i+1} , respectively.

The path connection and tool retraction strategy. The cutting path of each SM node is connected by the sequence after HM optimization, in which the transition path between two connected SM nodes is directly connected between two endpoints while a retraction path can be inserted between two SM nodes that are not connected in the graph (He et al., 2023).

4. Experimental results and discussions

In this section, simulation and physical experiments are reported on three exemplary complicated parts with deep cavities: tee pipe, blade with holes and conformal cooling channels, to validate the generality and effectiveness of the proposed methodology, which is shown by the workflow in Fig. 22.

Specifically, the sequence planning and tool path generation algorithms are implemented in C++ and executed on a PC equipped with an Intel Core i9-13900KF CPU @ 3.00 GHz, 128 GB RAM. The DED paths

of a group of AM nodes $\{N^a\}$ are converted into the G-code for DED, which is named like 'Part_0_A.NC' (where '0' stands for the total index of the NC file and 'A' denotes AM process). Similarly, the cutting paths of a group of SM nodes $\{N^s\}$ are converted into the G-code for machining, which is named like 'Part_1_S.NC'. The arranged A and S NC files are input to the software VERICUT® (64) v9.1.1 for simulation in sequence. In both the AM and SM simulation processes, if the collision is not reported in the software, the numerical control (NC) files will then be transmitted to the physical machine tool (JIATIE, KEJIE Technology, Fig. 22(d)), where two independent Z1 and Z2 axes are used for cutting and LP-DED, respectively. The parameters of DED and cutting are listed in Table 1 unless otherwise stated. For cases involving varied heights and laser power, the adaptive planning of parameters for multi-axis LP-DED is presented in the supplementary material.

4.1. Establishing the HM simulation environment using VERICUT

Fig. 23 shows the simulation environment using VERICUT, including the machine tool's fixture/base, controller, tool library (including nozzle), and NC files, which are all consistent with their counterparts in the physical space. As shown in Fig. 23(a), a typical line of AM G-code is G01 X_Y_Z_B_C_I_J_F_, in which the first three represent the position of the DED point, B & C denote the nozzle axis, I & J represent the laser power and focal length, respectively, and F is the scan speed of DED. In the AM simulation, the diameter of the DED bead d_b is calibrated

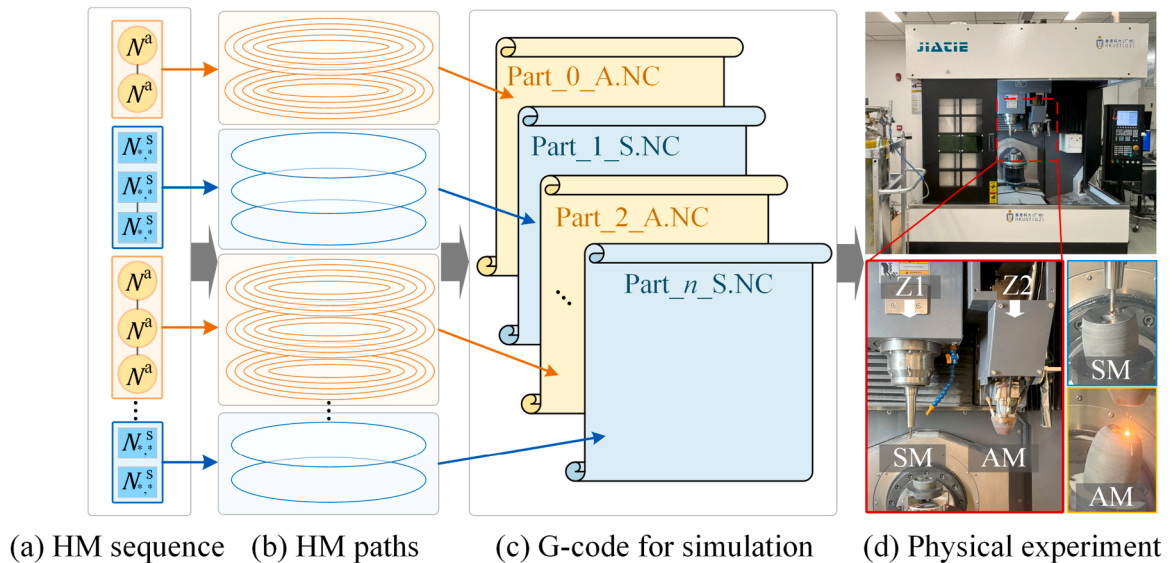


Fig. 22. The verification pipeline from (a) HM sequence, (b) paths, (c) simulation to (d) experiment on a five-axis HM center with two independent Z-axes: Z1 for cutting and Z2 for LP-DED.

Table 1
Parameters of DED and machining.

Parameters	Values
Five-axis machine tool	JIATIE, BC table
Powder material	Stainless steel 316 L
Powder flow rate (g/min)	7.66
Gas flow rate (L/min)	5
Laser power (W)	650 ~ 800
DED scan speed (mm/min)	372 ~ 994
Layer height interval (mm)	0.3 ~ 0.8
Laser beam diameter (mm)	2.1
Cutter type	Ball-end
The radius of the cutter (mm)	3 and 2
Number of flutes	2
Spindle speed (rpm)	10000
Feed per tooth (mm)	0.01
Cutting width (mm)	0.3 ~ 0.8
Cutting depth (mm)	0.5
Cutter material	Cemented carbide (with Balinit coating)

according to the scanned deposited morphology (Zhang et al., 2025) (see the inset of Fig. 23(a)). Considering the bead width, the AM boundary paths (Fig. 19) are first offset inwards by $d_b/2$ and then outwards by ϵ (say 0.5 mm) to generate the as-built subpart with the allowance thickness of ϵ . This residual material is further removed by the following SM in the A-S alternation. A typical line of SM G-code is G01 X_ Y_ Z_ B_ C_ F_ whose meaning is the same as AM, as shown in Fig. 23(b). The NC

files for AM and SM are imported by the A-S sequence for HM simulation, in which the deposited layers are machined, and the machined surface can be correctly displayed with correct collision detection, i.e., the IPW changes with time. As shown in the bottom-right inset of Fig. 23 (b), the surface morphology of the machined surface at ① is different from the layer pattern of the DED morphology at ②, which is consistent with the experimental result (Fig. 26 & Fig. 29), indicating the effectiveness of the simulation on HM.

Fig. 24 shows a concrete HM simulation instance of the tee pipe in Fig. 3(a), which is a typical multi-branch part partitioned and sliced into a set of AM (Fig. 3(b)) and SM nodes (Fig. 3(c)). The A-S alternation sequences are solved using the HM cost matrix in Fig. 9, and the IPWs for HM are shown in Fig. 24(a) to (f) in chronological order. Specifically, the bottom region of the pipe roughly consists of two segments: A1/S1 and A2/S2. The difference between the proposed method and the layer-based method is that the outside of the pipe remains unmachined (see the layered pattern of the inset of Fig. 24(a)) until the whole part is AM built, so to preserve the consistent machined morphology of the outer surface. After that, segment A3 is printed, spanning two branches Ω_1 and Ω_2 (see the different layered patterns of the inset of Fig. 24(c)), followed by the machining of S3. Segment A4 is then deposited on both branches, but only the inner side of the left one is machined since the right one will be machined together with the surface of A5 by a single SM operation (Fig. 24(g)). This asynchronism between AM and SM features the proposed method and helps to reduce the gear marks on the machined surface.

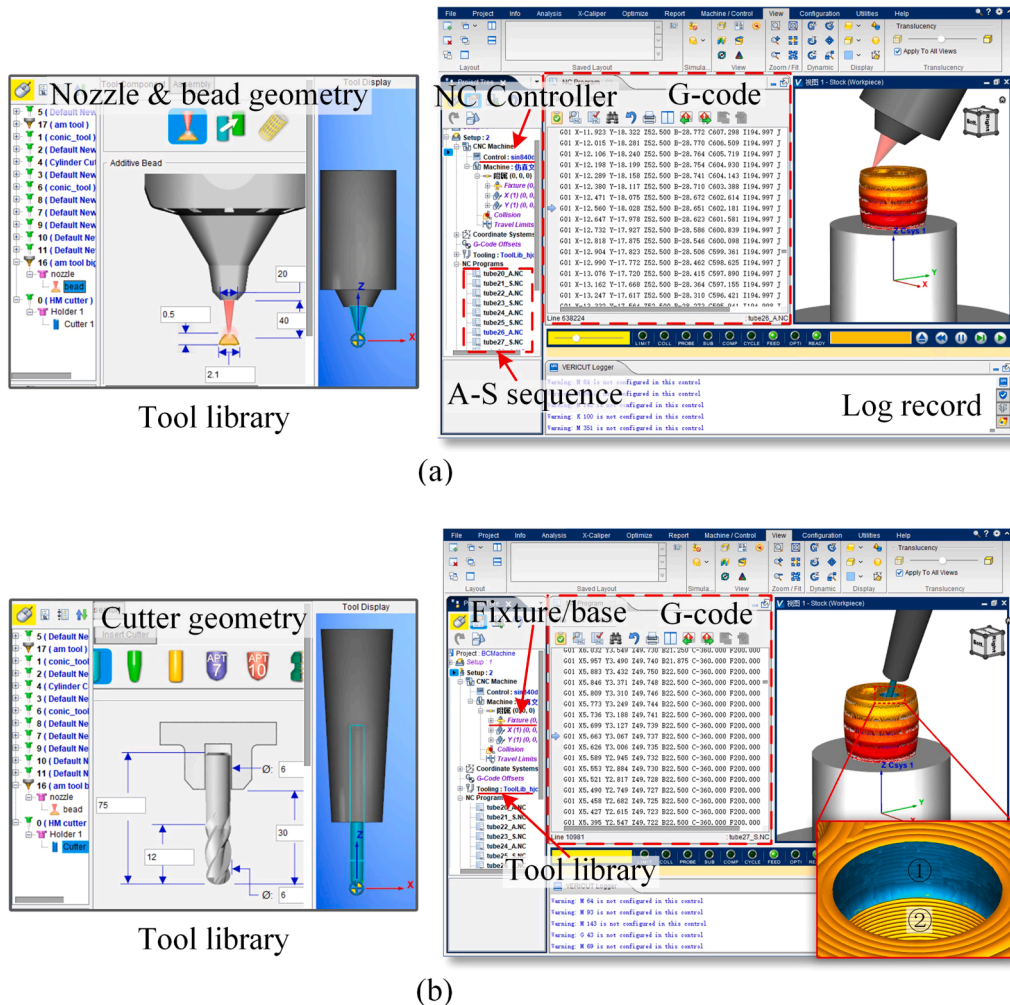


Fig. 23. Simulation of HM using VERICUT in which no collision of nozzle/cutter is reported by the software: (a) the multi-axis DED simulation, including the bead geometry setup; (b) the cutting simulation of the printed part.

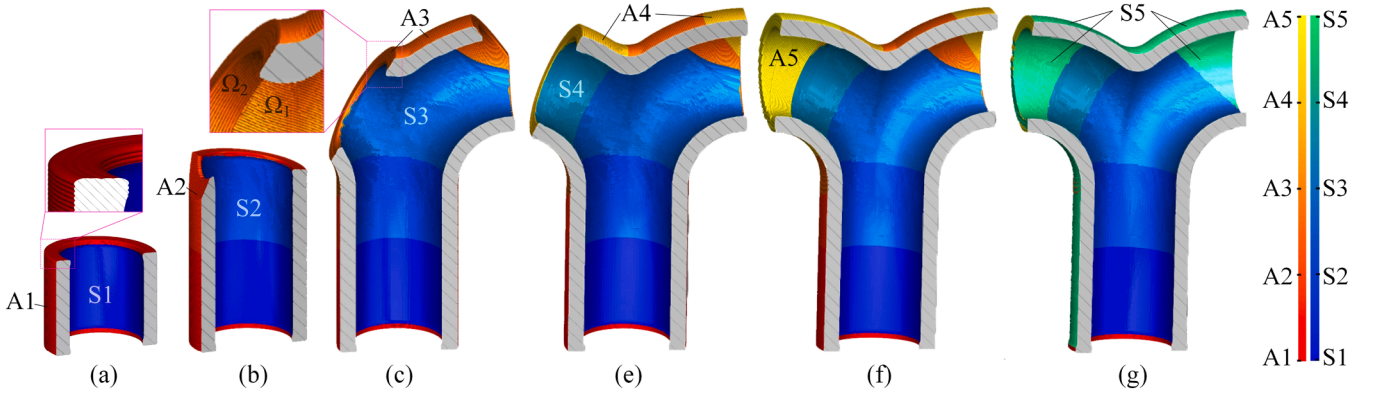


Fig. 24. The A-S alternation of the tee pipe in chronological order (a)~(g).

4.2. Comparison with the AM layer-based method

Fig. 25 shows another example – a blade with four holes of different sizes and shapes (Fig. 25(b)). The input model is sliced into 140 parallel Z-level planes with a thickness of 0.5 mm. The AM paths of one typical layer are shown in Fig. 25(a) which constitute a single AM node while each connected SM path corresponds to one SM node. In the layer-based alternating scheme (Chen et al., 2025), each segment is first printed and then machined subsequently, thus generating eight HM segments without considering the shape characteristics of each hole (Fig. 25(c)). On the contrary, the proposed method handles each hole independently in which the left three holes have seven SM segments while the rightmost one has eight (Fig. 25(d)). This is because the left three holes have almost vertical and straight shapes throughout the blade although the leftmost one is smaller in size. The rightmost one, however, is inclined in shape, so the collision situation becomes harsh, and its uppermost region is split into two SM segments, i.e., S7 and S8. Thus, the proposed method is able to reduce the SM segment by one for the three holes in the left, thereby alleviating the staircase effect so as to improve the surface

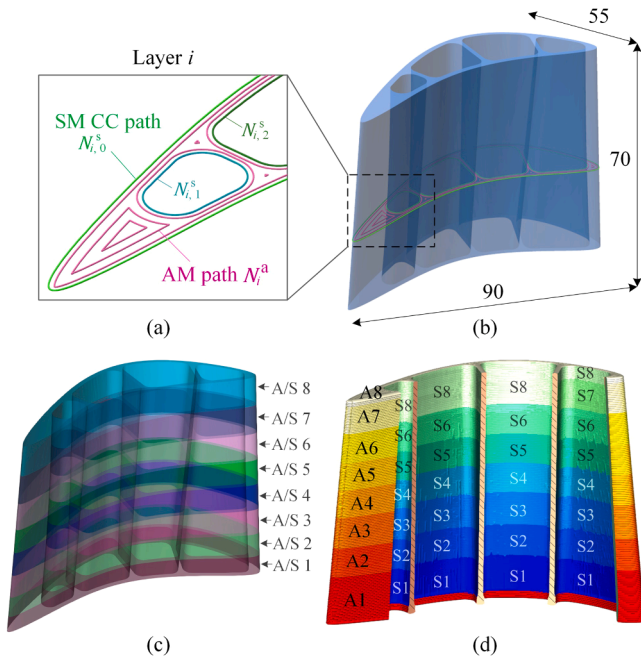


Fig. 25. Validation of the blade with four holes: the AM paths of one sliced layer in (a) extracted from (b) constitute 1 AM node while each connected SM path corresponds to one SM node. The layer-based sequence planning method in (c) requires eight A-S alternations for all the holes but the proposed method in (d) reduces one alternation for three holes.

quality of the inner wall. In addition, due to the large solution space of the SOP-based HM planning, there is no unique segmentation strategy. The top-down search method tends to generate relatively large HM segments on the top region while the proposed method is able to uniformize the heights of each segment thanks to the higher searching ability of the LKH algorithm.

The simulation of the HM sequence is detailed in the insets of Fig. 26, in which the four holes are machined synchronously from Fig. 26(a) to (f) until S7 is machined only at the rightmost hole, followed by Fig. 26 (g)&(h). The number of alternations in the left three holes is seven, representing a 12.5% reduction compared to the layer-level method (which has eight alternations) (Chen et al., 2025). This sequence is consistent with the physical HM experiment where the upper plane of the IPW after each AM operation has to be machined off in order to maintain the same deposition height as that in the G-code. Thus, the burrs after cutting in Fig. 26 can reflect the AM sequence and the machined inner surface can be clearly identified by the cutting texture (which is different from the DED surface morphology). To demonstrate the advantage of HM over the conventional pure machining, the percentage of the removed material using HM is calculated (39.4%) which is compared with the pure machining (90.7%), indicating a sharp reduction (57%) in material waste by HM.

The advantages of the proposed path-level sequence planning method over the layer-level one can also be reflected from Fig. 27(b)&(c). For the left straight channel, the number of alternations decreased from 6 to 4 (a 33.3% reduction) compared to the previous layer-level method (Fig. 27(b)). When considering the remelting avoidance effect, the alternations decreased from 7 to 5 (a 28.6% reduction in Fig. 27(c)).

4.3. Effects of the remelting layers

In this subsection, the proposed method is validated through both simulation and experimental fabrication of two conformal cooling channels featuring distinct curvatures (Fig. 27(a)). As shown in Fig. 27 (b), the proposed path-level method can handle HM sequence planning of two channels independently by generating different SM segments – the left channel Ch₁ is straight and the collision is alleviated, so four SM segments are sufficient (S1, S3, S4, and S6), meaning that S3 is performed after both A2 and A3 operations are finished. In contrast, as the right one Ch₂ is severely bent, the inner surface S2 has to be machined immediately after A2 is deposited, which creates a different HM pace between the two channels. This A-S alternation between multi-axis cutting and multi-axis DED is shown in Fig. 22(d).

This different HM pace between Ch₁ and Ch₂ can be reflected in the cost matrix C from Fig. 28 which shows a similar structure as that in Fig. 9. In specifics, referring to the sub-matrix $C_{12}^{n_A \times n_S}$, each column represents a CC path belonging to one of three regions: the outer surface, Ch₁ or Ch₂ (see the enlarged view of ③), in which three adjacent columns correspond to 1 AM layer, say N_i^A . The collision-free constraints

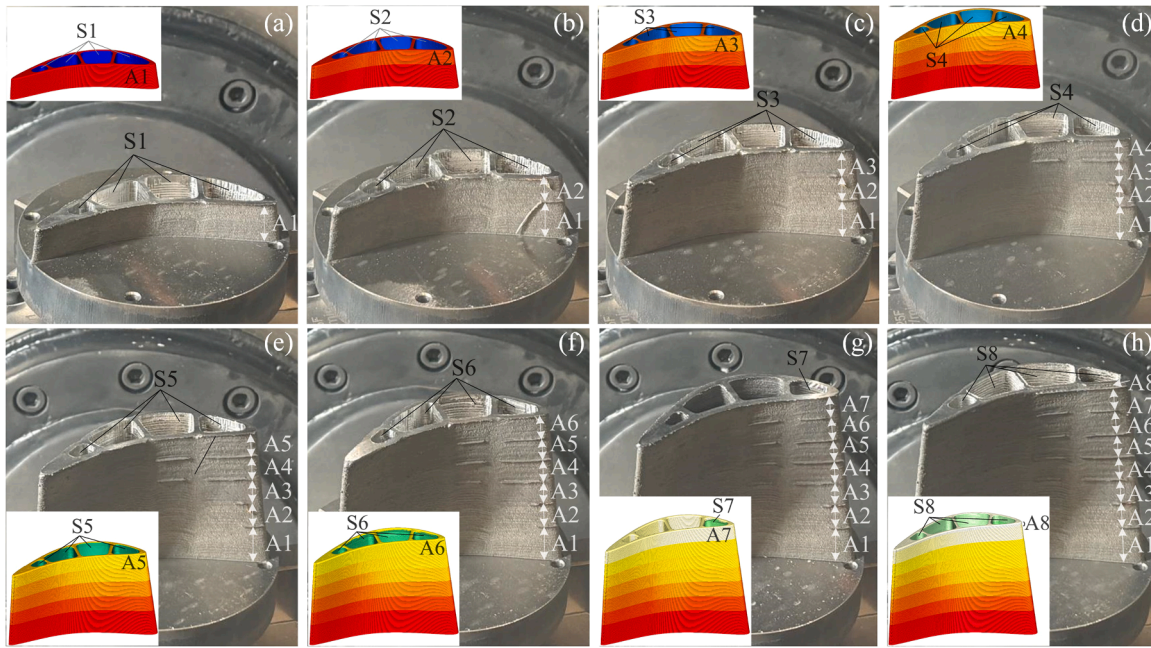


Fig. 26. The A-S alternation of the simulation and physical verification on the blade considering the remelting avoidance, in which the IPWs are shown in chronological order (a)~(h).

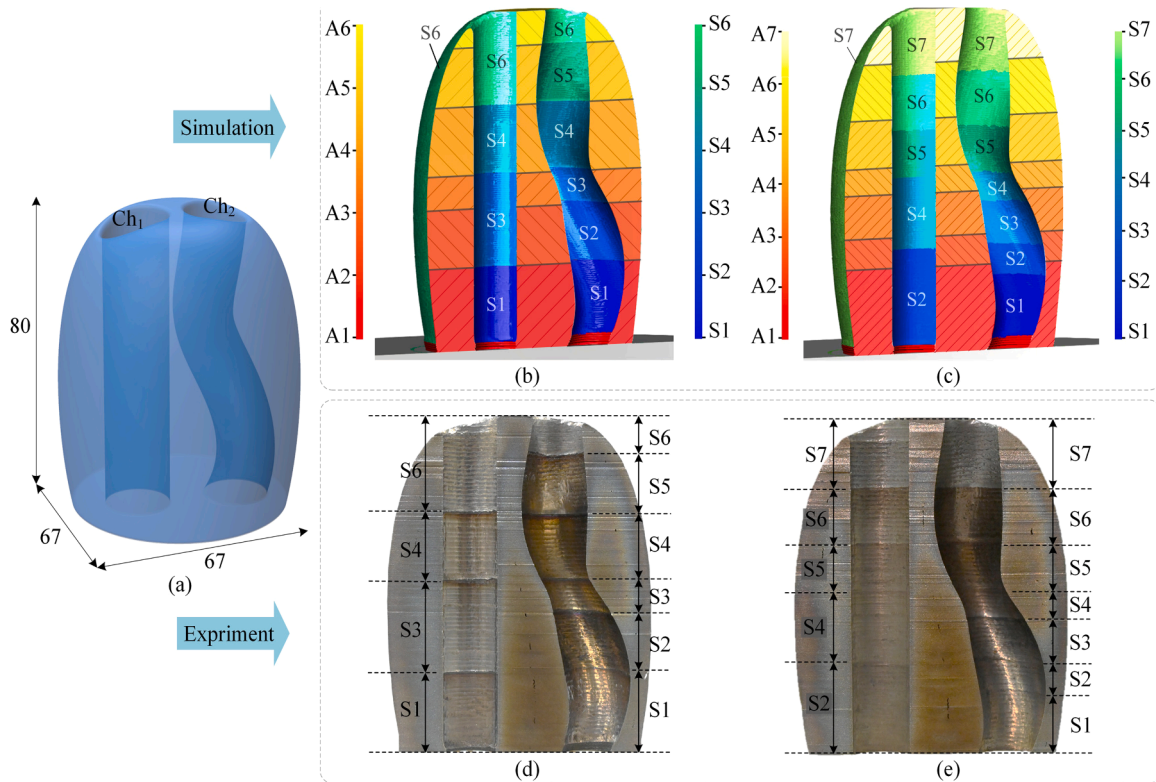


Fig. 27. The validation on (a) two conformal cooling channels with different shapes; the section view of the final part after HM simulation (b) & experiment (d) without the remelting avoidance; the section view of the final part after HM simulation (c) & experiment (e) with the remelting avoidance.

are labeled by $C(i,j) = -1$ when machining SM node N_j^s with AM node N_i^a being the obstacle. Overall, the number of “-1” elements in $C_{12}^{n_A \times n_s}$ shows a decreasing trend as the SM nodes’ indices become larger, indicating fewer AM layers will collide with the SM node. However, these three regions demonstrate different behaviors in terms of A-S collision constraints: for the columns corresponding to the outer surface,

no “-1” elements exist, meaning all the SM nodes of the outer surface can be machined totally collision-free after the whole part is manufactured; the SM nodes of Ch₁ have fewer “-1” elements than those of Ch₂ from the same AM layer, due to the straight shape of Ch₁, indicating fewer SM segments will be generated.

When integrating the remelting avoidance constraints in [Section](#)

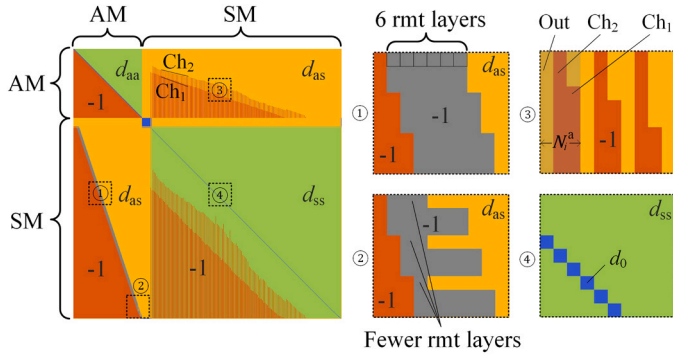


Fig. 28. The cost matrix C of the conformal cooling channels whose enlarged views are shown on the right.

3.3.1 (Fig. 27(c)), both AM and SM segment counts increase by one compared with those in Fig. 27(b). This is because more precedence constraints like the ones in Fig. 20(e) are imposed on the HM digraph. These extra constraints can also be reflected in the cost matrix C from Fig. 28. Essentially, the extra remelting avoidance constraint tightens the dependency preservation constraint in the sub-matrix $C_{21}^{n_s \times n_a}$, by adding extra n_{rmt} elements of “-1” for each SM node, as illustrated by the inset ①. To avoid the A-S interlock issue in Section 3.3.4, the number of remelting layers has to be decreased, as shown in the inset ②. In order to ensure that CC paths are machined in the reverse order of the printing, asymmetric costs are assigned between two adjacent SM nodes as d_0 and d_{ss} (see the inset ④). The HMed parts after wire electrical discharge machining are shown in Fig. 27(d)&(e) which are consistent with the simulation: the machined pattern of the inwall demonstrates the accessibility of the proposed method under the A-S alternation scheme; the one without considering the remelting avoidance (Fig. 27(d)), which has fewer alternations, shows severe oxidation color near the A-S interfaces, while the oxidation color for the one considering the remelting avoidance is much alleviated (Fig. 27(e)).

The HM sequence validation considering the remelting avoidance is

illustrated in Fig. 29 in which the simulation in the insets coincides well with the experiment. In the experiment, like Fig. 26, after each AM process, the over-deposited material exceeding the upper plane of the IPW is removed such that the machined part has the same height as the design shape.

4.4. CT and laser scan of the parts after HM

To quantitatively assess the surface quality and deviation of the HMed part, a computed tomography (CT) scan (YXLON) is performed on the manufactured blade model (Fig. 26). The inner walls are extracted, registered, and compared with the CAD model. Fig. 30(a) shows the reconstructed CT volume data of the blade (with holes), while the inner walls are displayed in the section view of Fig. 30(b). Although the machining patterns are noticeable, its scallop height can be reduced by shortening the step-over distance by inserting SM nodes between two consecutive ones, as stated in Section 3.4. The model registration indicates that, after machining, the geometry obtained is in general consistent with the CAD model, including the bumped A-S interfaces in Fig. 30(b) which are consistent with the virtual counterpart of Fig. 25 (d). The deviation between the CT-scanned and CAD models is evaluated (the maximum is found to be around 0.8 mm), in which the discontinuity of the inner surface can be observed which stems from the repeated heating and machining at the A-S interfaces. The local HM deformations of holes 1 & 4 are typically smaller than those of holes 2 & 3 due to their larger stiffness (smaller size).

In order to characterize the surface morphology of the inwall, the geometry of the wired-cut parts in Fig. 27(d)&(e) are scanned using a laser tracker (Leica AT960), and the scanning results are shown in Fig. 31. For roughness measurement, the surface roughness at vertex i is quantified by the averaged angular difference between i and its neighbor vertices j , i.e., $\gamma_i = \sum_{j \in \mathcal{N}(i)} \text{acos}(\mathbf{n}_i^T \cdot \mathbf{n}_j^T) / N$. The colormap in Fig. 31 shows the γ distribution across the internal walls, where the one without the remelting avoidance in (a) exhibits a significant local bump near the theoretical A-S interfaces while the one incorporating the remelting avoidance in (b) shows significantly reduced staircase effect, which is

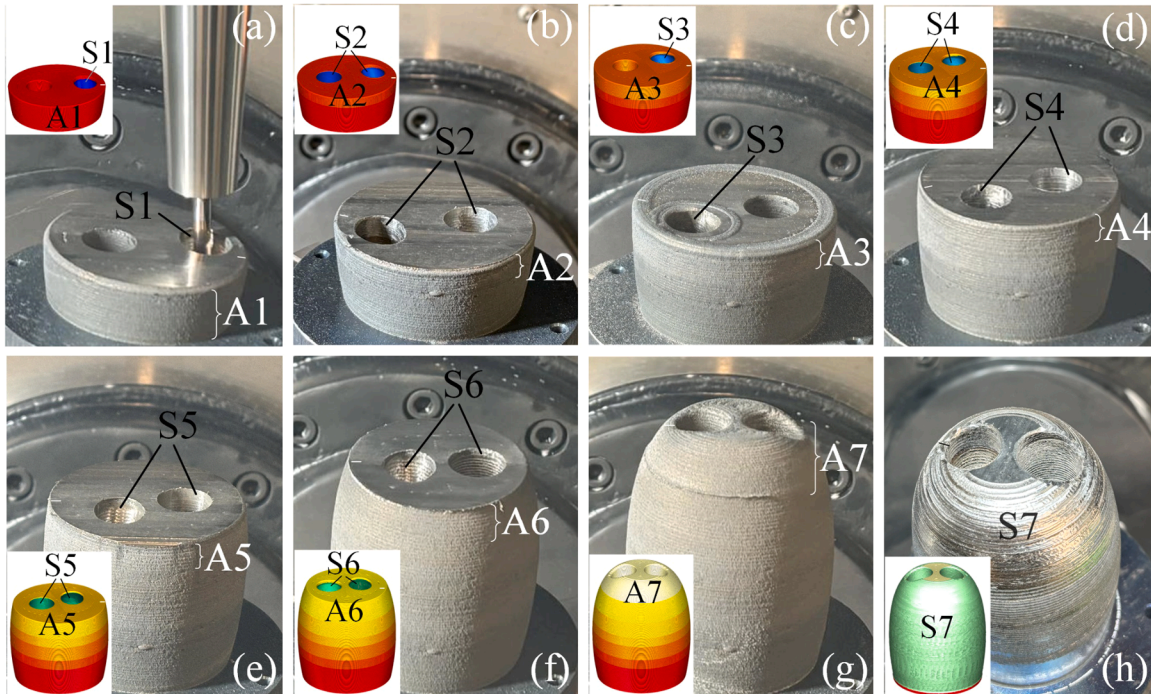


Fig. 29. The A-S alternation of the simulation and physical verification on the conformal cooling channels considering remelting avoidance, in which the IPWs are shown in chronological order (a)~(h).

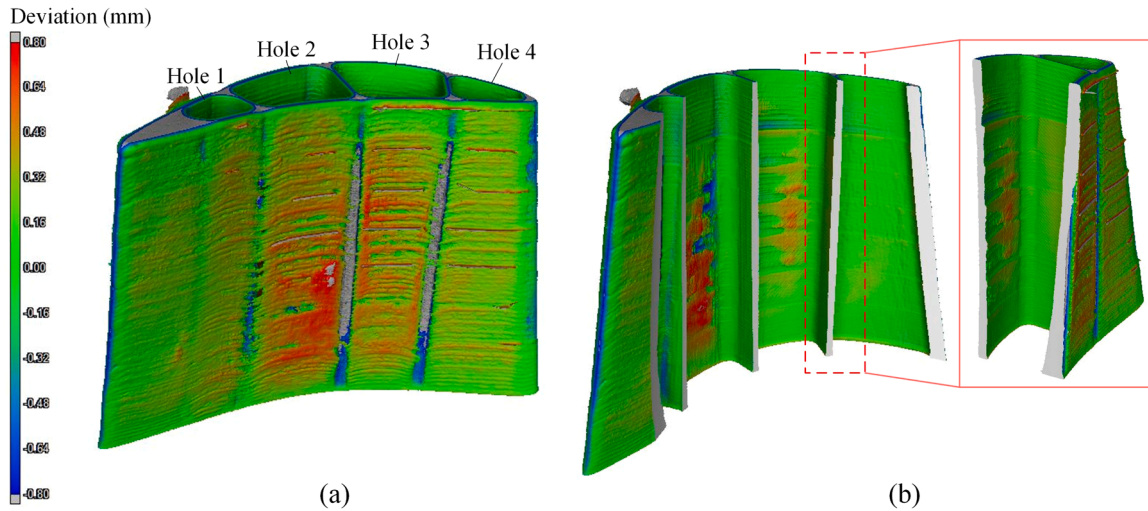


Fig. 30. The CT scan of the blade: (a) the reconstructed volume with the deviation represented by the colormap; (b) the section view of (a).

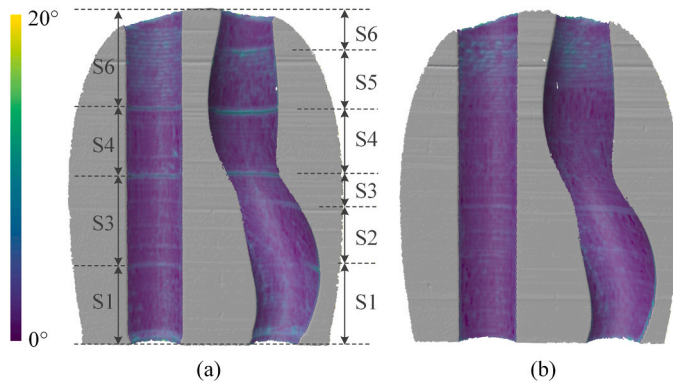


Fig. 31. The laser-scanned mesh of the wired-cut channel model in Fig. 27(d)& (e) with the surface roughness distribution of the inwall: (a) without remelting avoidance; (b) with remelting avoidance.

consistent with the assumptions.

5. Conclusions

This study presented a comprehensive multi-axis HM sequence planning framework for complex components featuring multiple deep cavities, which optimizes the path-level A-S (additive-subtractive) alternation sequence under the collision-free and other physical property-related constraints to enhance surface finish quality and manufacturing efficiency. Faced with the chicken-and-egg problem between the uncertainty of cutter axes and multi-axis HM sequence planning, it is demonstrated that the sequence optimization for multi-axis HM can be formulated as an SOP built upon the optimization of a sub-problem for pure AM, in which the HM cost matrix $C^{(n_A+n_S) \times (n_A+n_S)}$ encompasses all the dependency and collision-free constraints as well as the HM preferences. Simulation and experimental results on exemplary parts indicate that the proposed path-level HM sequence planning method improves surface finish quality over conventional layer-level approaches by reducing machining segments by up to 30%. Compared with pure machining, the HM for the blade part can reduce the material waste by up to 57%.

As for the future work, firstly, in this paper, when calculating the IPW for collision detection, the deformation induced by DED and cutting is

not considered, although the volume of over-deposited material is small. For a more accurate description of the IPW's shape, its as-built shape should be used. Secondly, due to the inherent nature of LP-DED, the support structure is still required for extremely complicated modes, especially with a non-zero genus structure. In this case, to remove the support of each layer, the CC paths of any SM node must have multiple contours that are independent of the AM paths. Fortunately, this decoupling between AM and SM can be integrated into the proposed framework conveniently by modifying the weights and constraints of the HM cost matrix whose construction strategy will be the future work. Finally, the number of remelting layers in this paper is specified as a fixed number, and the modelling of metallurgy that relates the number of remelting layers to the properties could be integrated to enhance the performance, which will be the future work as well.

CRediT authorship contribution statement

Pengcheng Hu: Resources, Methodology, Funding acquisition. **Yuanzhi Chen:** Software, Data curation. **Molong Duan:** Resources, Funding acquisition. **Jiejun Xie:** Validation, Software. **Kai Tang:** Writing – review & editing, Supervision, Funding acquisition. **Xiaojun Liu:** Resources, Funding acquisition. **Jiancheng Hao:** Validation, Software, Investigation, Data curation. **Dong He:** Writing – original draft, Validation, Software, Methodology, Conceptualization. **Lulu Huang:** Visualization, Validation, Data curation. **Tak Yu Lau:** Visualization, Methodology, Data curation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix: Glossary

AM	Additive manufacturing
SM	Subtractive machining
SOP	Sequential Ordering Problem
Ω	Input watertight triangular mesh model
L_i	i -th AM layer (basic unit for AM)
N_i^a	i -th AM node
S_i	i -th bottom slice of AM layer L_i
S_i^{i+1}	The lateral surface sandwiched between S_i and S_{i+1}
S_p	The lateral surface inherited from the partition planes
$[\delta t_{\min}, \delta t_{\max}]$	Thickness range
$\partial S_{i,j}$	j -th SM path for layer L_i
$N_{i,j}^s$	j -th SM node for layer L_i
$\mathcal{T}_a(\mathbf{p}, \mathbf{t})$	AM nozzle assembly located at $\mathbf{p}$ with axis $\mathbf{t}$
$\mathcal{T}_s(\mathbf{p}, \mathbf{t})$	Cutter assembly located at $\mathbf{p}$ with axis $\mathbf{t}$
d_0	Zero-cost edge for adjacent AM/SM nodes
d_{aa}, d_{ss}	Air movement cost for non-adjacent AM/SM nodes
d_{as}	Large cost A-S alternation
$\mathbf{C}$	Cost matrix for SOP
n_A, n_S	Number of AM/SM nodes in the digraph
$\{N_{pre}^a\}_i$	Previous AM nodes of N_i^a in the AM digraph
$\mathbf{p}_{cc}$	Cutter contact (CC) points
$\mathbf{a}$	Nozzle axis vector
$\mathbf{n}_f$	Surface normal at $\mathbf{p}_{cc}$
α_{lim}	The maximum allowed angle of the nozzle axis
$\mathcal{F}$	Fixture assembly
θ^*	Preferred angle of nozzle axis relative to $\mathbf{n}_f$
$f(\mathbf{a}; \mathbf{p}_{cc})$	Feasibility index of axis $\mathbf{a}$ at $\mathbf{p}_{cc}$
$\mathcal{R}(\mathbf{a}; \mathbf{p}_{cc})$	Remainder term for tool axis preference
$w(\mathbf{a}_j^i, \mathbf{a}_k^{i+1})$	Weight between $\mathbf{a}_j^i$ and $\mathbf{a}_k^{i+1}$ for axis smoothing
λ	Balancing term between the smoothness and feasibility
$\mathcal{A}$	The area of the cross-section of a layer
n_{rmt}	The number of layers affected by the remelting
$\mathcal{D}(N_i^a, N_r^a)$	The search depth from N_i^a to N_r^a
$\mathcal{T}(\mathbf{p})$	The user-defined forbiddance map at $\mathbf{p}$
$\tilde{d}_{as}$	The modified cost d_{as}
$\mathcal{N}$	The number of points of N_i^a
$\mathcal{H}_i^{i+1}$	The local cusp height $\mathcal{H}_i^{i+1}$ between $N_{i,j}^s$ and $N_{i+1,j}^s$
$\mathbf{u}$	The feed direction at $\mathbf{p}$
$\delta t(\mathbf{p})$	The local step-over distances $\delta t(\mathbf{p})$ at point $\mathbf{p}$
κ_s	Normal curvature of the surface along step-over direction
κ_c	Curvature of the ball-end cutter
Π_i	i -th slicing plane
$\mathbf{n}_i$ and $\hat{\mathbf{p}}_i$	The normals and anchor points of Π_i

Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at [doi:10.1016/j.compind.2026.104493](https://doi.org/10.1016/j.compind.2026.104493).

Data availability

Data will be made available on request.

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